

# Electroweak radiative corrections in Z boson decays

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## **Bibliography.**

# 1 Introduction.

## New theories, new symmetries, new particles, new phenomena.

The creation of the unified electroweak theory at the end of the 1960s [1], [2] and of the quantum chromodynamics at the beginning of the 1970s [3] has dramatically changed the entire picture of elementary particle physics. Its foundation changed to gauge symmetries: electroweak symmetry  $SU(2)_L \times U(1)$  and the color symmetry  $SU(3)_c$ . It became clear that the gauge symmetries determine the dynamics of the fundamental physical processes in which the key players are the gauge vector bosons—the well-familiar photon and a host of new particles:  $W^+$ ,  $W^-$ ,  $Z$  bosons and eight gluons which differ from one another in color charge. Even though the Higgs condensate, filling the entire space, remains enigmatic in the electroweak theory, while the problem of confinement is still unsolved in chromodynamics, the two theories are nevertheless so inseparable from modern physics that they were given the name of the Minimal Standard Model (MSM). We assume in this review that the reader is familiar with the basics of the MSM (see, for example, the monographs [4]).

The concept of quarks has undergone dramatic expansion in the process of creation of the MSM. In [1] the electroweak theory was suggested for leptons (electron and electron neutrino). The subsequent inclusion of quarks into the theory led to the hypothesis that in addition to the three quarks known at the time— $u$ ,  $d$  and  $s$ —there exists the fourth quark,  $c$ . According to [2], if  $d$ - and  $s$ -quarks are analogues, respectively, of  $e$  and  $\mu$ , then the mutually orthogonal combinations  $u \cos \theta_c + c \sin \theta_c$  and  $-u \sin \theta_c + c \cos \theta_c$ , where  $\theta_c$  is the Cabibbo angle, must constitute the analogues of  $\nu_e$  and  $\nu_\mu$  [5].

One of the important consequences of the electroweak theory was the prediction of the weak neutral currents. According to the theory, the neutral weak currents must be diagonal; in other words, neutral currents changing quark flavor (FCNC) are forbidden. This explained the absence of such decays as  $K^0 \rightarrow \bar{e}e$ ,  $K^0 \rightarrow \bar{\mu}\mu$  and  $K^+ \rightarrow \pi^+ \bar{e}e$ . Since there is no neutral current  $\bar{s}d$  in the Lagrangian, these decays cannot occur in the tree approximation: they require loops with virtual  $W$ -bosons. This is also true for the transitions  $K^0 \rightarrow \bar{K}^0$  that are responsible for the mass difference between the  $K_L^0$  and  $K_S^0$ -mesons.

Diagonal neutral currents were discovered in reactions with neutrino beams [6], in rotation of the polarization plane of a laser beam in bismuth vapor [7], and in the scattering of polarized electrons by deuterons [8].

The charmed quark  $c$  was discovered in 1974 [9]. Even before that, Kobayashi and Maskawa [10] conjectured that in addition to two generations of leptons and quarks,  $(\nu_e, e, u, d)$ ,  $(\nu_\mu, \mu, c, s)$ , there must exist the third generation  $(\nu_\tau, \tau, t, b)$ . The  $2 \times 2$  Cabibbo matrix for two generations,

$$\begin{pmatrix} \cos \theta_c & \sin \theta_c \\ -\sin \theta_c & \cos \theta_c \end{pmatrix}$$

is replaced in the case of three generations by a  $3 \times 3$  unitary matrix that in its most general form contains three angles and one phase; the phase is nonzero if the CP-invariance is violated. This is how the mechanism of CP violation at the level of quark currents was proposed.

The  $\tau$ -lepton [11] and the  $b$ -quark [12] were discovered experimentally in mid-1970s. The

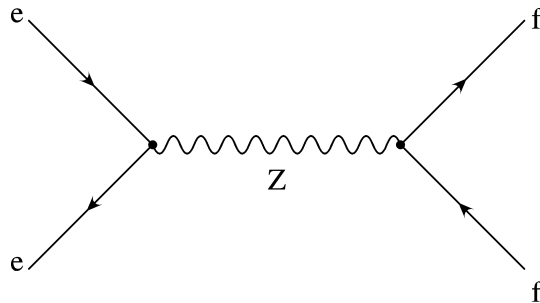


Figure 1: The  $Z$  boson as a resonance in  $e^+e^-$  annihilations. A fermion–antifermion pair in the  $Z$ -boson decay is denoted by  $f\bar{f}$ , where  $f$  can be either a lepton ( $e, \mu, \tau, \nu_e, \nu_\mu, \nu_\tau$ ), or a quark ( $u, d, s, c, b$ ). In this last case a quark–antiquark pair typically transforms, owing to interactions with gluons, to a multi-hadron state. The outgoing arrow in this and subsequent diagrams corresponds to the emission of the fermion ( $f$ ) and to absorption of the antifermion ( $e^+$ ); an incoming arrow denotes the emission of the antifermion ( $\bar{f}$ ) and the absorption of the fermion ( $e^-$ ).

heaviest fermion—the  $t$ -quark—was discovered only two decades later [13] [14]. As for the mechanism of CP-invariance violation, it remains unknown even now.

The renormalizability of the electroweak theory and quantum chromodynamics [15] and the property of asymptotic freedom in QCD [16] opened a wide field for reliable computations based on perturbation theory. On the basis of such computations in the tree-diagram approximation, it was possible to predict such qualitatively novel phenomena as quark and gluon jets; using the data on neutral currents, it proved possible to perform preliminary computations of the masses and partial widths of the  $W$  and  $Z$  bosons even before their actual discoveries.

### W and Z boson ‘factories’.

To test the predictions of the electroweak theory, proton–antiproton colliders were built at the beginning of the 1980s in Europe (at CERN) and then in the USA (at FNAL). The discovery of the  $W$  and  $Z$  bosons [17] provided spectacular confirmation of the tree-diagram calculations [4] and made feasible and urgent precision tests of the electroweak theory with loops included. (When speaking about the tree (Born) approximation and loops, we always mean the corresponding Feynman diagrams.)

A unique object for such tests was the  $Z$  boson. To carry out a precision study of its properties, electron–positron colliders were built at the end of the 1980s: LEP1 at CERN and SLC at SLAC. Electron and positron in these colliders collide at the center-of-mass energy equal to the  $Z$  boson mass. This results in the resonance creation and decay of a  $Z$  boson (see figure 1). The LEP1 completed its operations in October 1995; about 20 million  $Z$  bosons were detected in the four detectors of this collider (ALEPH, DELPHI, L3, OPAL). The total number of  $Z$  bosons recorded at the SLC by the sole SLD detector was approximately  $10^5$ ; however, since the colliding electrons are longitudinally polarized, it was

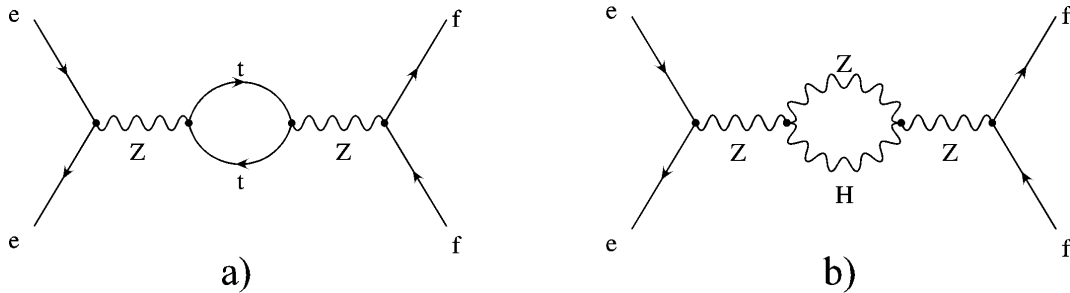


Figure 2: Contribution of the  $t\bar{t}$  to the  $Z$ -boson propagator (a). Contribution of the loop with virtual  $Z$ -boson and a higgs to the  $Z$ -boson propagator (b).

possible to study the dependence of the annihilation cross section  $e^+e^-$  of the  $Z$  boson on the sign of beam polarization. As a result, the SLC proved its competitiveness even with substantially lower statistics. The statistical and systematic accuracy achieved in the study of the properties of the  $Z$ -boson are of the order of  $10^{-5}$  for the  $Z$  boson mass and of the order of several thousandths for the observables that characterize its decays.

### What's the point of studying loop corrections?

A natural question is: why do we need to compare the experimental data and the loop corrections of the electroweak theory? We need it mostly to gather data on the not yet discovered particles. For instance, even before the  $t$ -quark was discovered on the Tevatron by CDF [13] and D0 [14] collaborations, its mass was predicted by analyzing the radiative loop corrections and the LEP1 and SLC data [18]. The main loop involving the virtual  $t$  and  $\bar{t}$ -quarks is shown in figure 2a. The scalar Higgs boson (or simply the higgs) had not been found yet. In the minimal version of the theory, the so-called Minimal Standard Model (MSM), there is a single higgs—a neutral particle whose mass is not fixed by the model. In the Minimal Supersymmetric Standard Model (MSSM) we have three neutral and two charged higgses. The lightest of the neutral higgses must not be heavier than 135 GeV [19], [20]. The simplest diagram involving a virtual higgs is shown in figure 2b.

When planning experiments on LEP1 and SLC, people had great expectations that precision measurements would detect pronounced deviations from the predictions of the standard model and would thereby unambiguously point to the reality of some sort of ‘new physics’. In fact, even though some discrepancies with the MSM were found, they go beyond the three standard deviations only in a single case (that of the decay of a  $Z$  boson to a  $b\bar{b}$ -pair). If these discrepancies are not caused by some sort of systematic error, they may indicate (see Conclusions) the existence of the relatively light ( $\sim 100$  GeV) squarks and gluino—the supersymmetric partners of quarks and gluons, respectively.

## 2 Brief history of electroweak radiative corrections.

The pioneer calculations of electroweak corrections in MSM were performed in the 1970s, long before the discovery of the  $W$  and  $Z$  bosons. The calculations were devoted to the muon decay and the  $\beta$ -decays of the neutron and nuclei and to deep inelastic processes. In connection with the construction of LEP and SLC, a number of teams of theorists carried out detailed calculations of the required radiative corrections. These calculations were discussed and compared at special workshops and meetings. The result of this work was the publication of two so-called ‘CERN yellow reports’ [21], [22], which, together with the ‘yellow report’ [18], became the ‘must’ books for experimenters and theoreticians studying the  $Z$  boson.

### Muon and neutron decays.

Sirlin [23] calculated the radiative corrections to the muon decay due to one-loop Feynman diagrams (see figure 3). We must emphasize that the purely electromagnetic correction to the muon decay due to the exchange of virtual photons and the emission of real photons was calculated even earlier [24], for the pointlike four-fermion interaction, i.e. without taking the  $W$ -boson into account (see figure 4). It was found that the correction is finite: it contained no divergences. The four-fermion interaction constant  $G_\mu$ , extracted from the muon lifetime  $\tau_\mu$ ,

$$\frac{1}{\tau_\mu} = \Gamma_\mu = \frac{G_\mu^2}{192\pi^3} m_\mu^5 f\left(\frac{m_e^2}{m_\mu^2}\right) \left(1 - \frac{\alpha}{2\pi} \left(\pi^2 - \frac{25}{4}\right)\right) \quad (1)$$

where  $f(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \ln x$ , already includes this electromagnetic correction proportional to  $\alpha$ .  $G_\mu = 1.16639(2)10^{-5} GeV^{-2}$ . The finiteness of the purely electromagnetic correction in the muon decay is caused by the V–A nature of the interacting charged currents  $\bar{\nu}_\mu \mu$  and  $\bar{e} \nu_e$ .

In the neutron decay, the purely electromagnetic correction to the four-fermion interaction (figure 4(j)) diverges logarithmically. In view of the  $W$ -boson propagator (figure 4(h)), the logarithmic divergence is cut off at the  $W$ -boson mass.

This correction to the vector vertex in the leading logarithmic approximation, calculated in [25], is given by the factor

$$1 + \frac{3\alpha}{2\pi} \ln \frac{m_W}{m_p} \quad , \quad (2)$$

where  $m_p$  is the proton mass. Numerically, its value is of the order of 1.7%. Only after this correction is taken into account, does  $\cos^2 \theta_c$  extracted from the nuclear  $\beta$ -decay become equal to  $1 - \sin^2 \theta_c$ , where  $\sin^2 \theta_c$  is found from the decays of strange particles ( $\theta_c$  is the Cabibbo angle). Although the correction we discuss now contains  $m_W$  in the logarithmic term, it is essentially electromagnetic and not electroweak, since it is insensitive to details of the electroweak theory at short distances, in contrast to, say, electroweak corrections to the muon decay (figure 3).

Calculations of electroweak corrections to the muon decay show that the main contribution, exceeding all others, is caused by the vacuum polarization of the photon (figure 5a). At the first glance, this correction should not have emerged in the  $\mu$ -decay in the one-loop approximation: it is not there among the loops in figure 3. It does appear, however, when

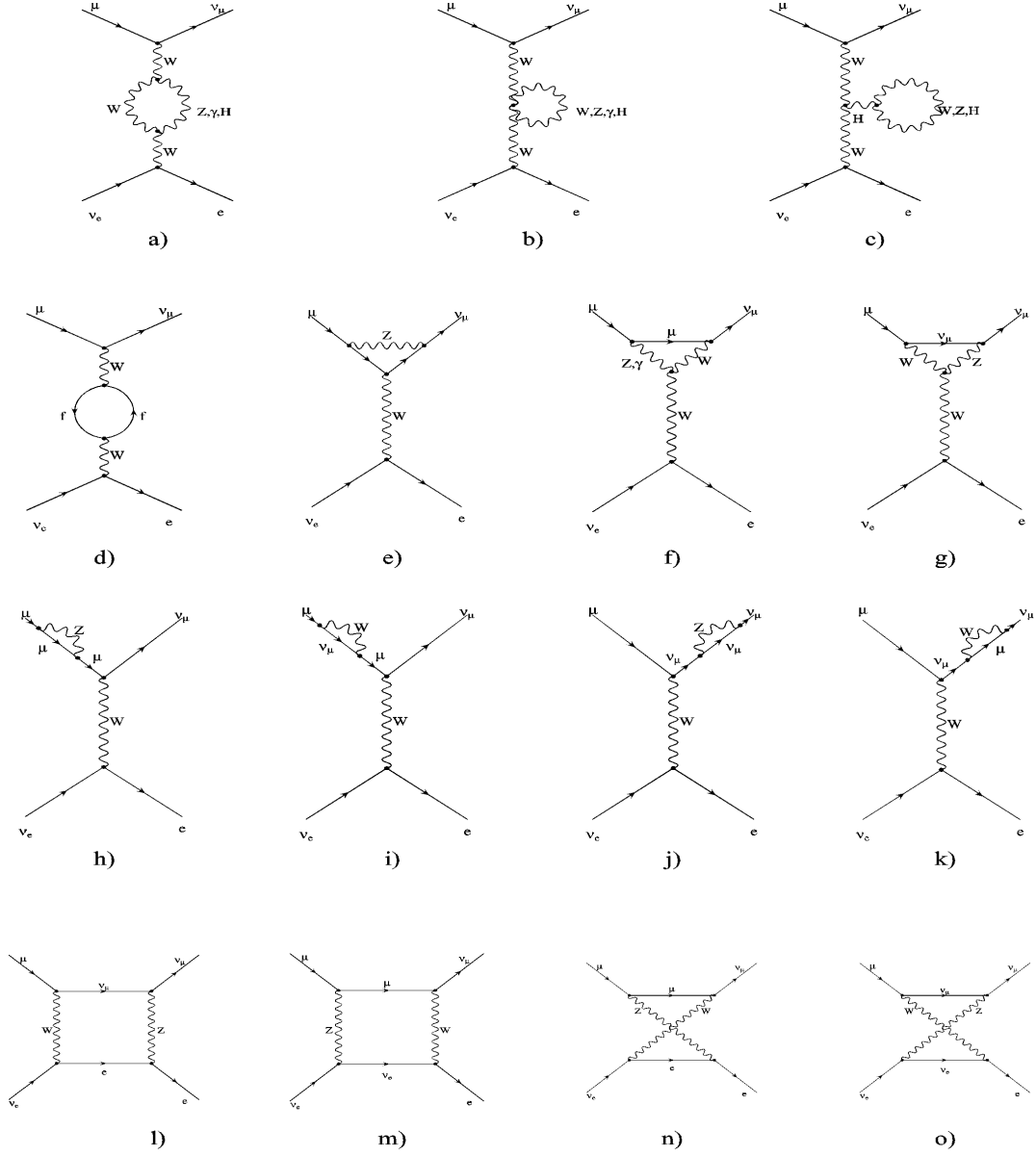


Figure 3: One-loop diagrams in muon decay. Loops in the  $W$ -boson propagator (a), (b), (c), (d); in the  $W$ -vertex (e), (f) and (g) and in external fermion lines (h), (i), (j), (k) (similar diagrams for  $e$  and  $\bar{\nu}_e$  are assumed), as well as square-type diagrams (l), (m), (n), (o).



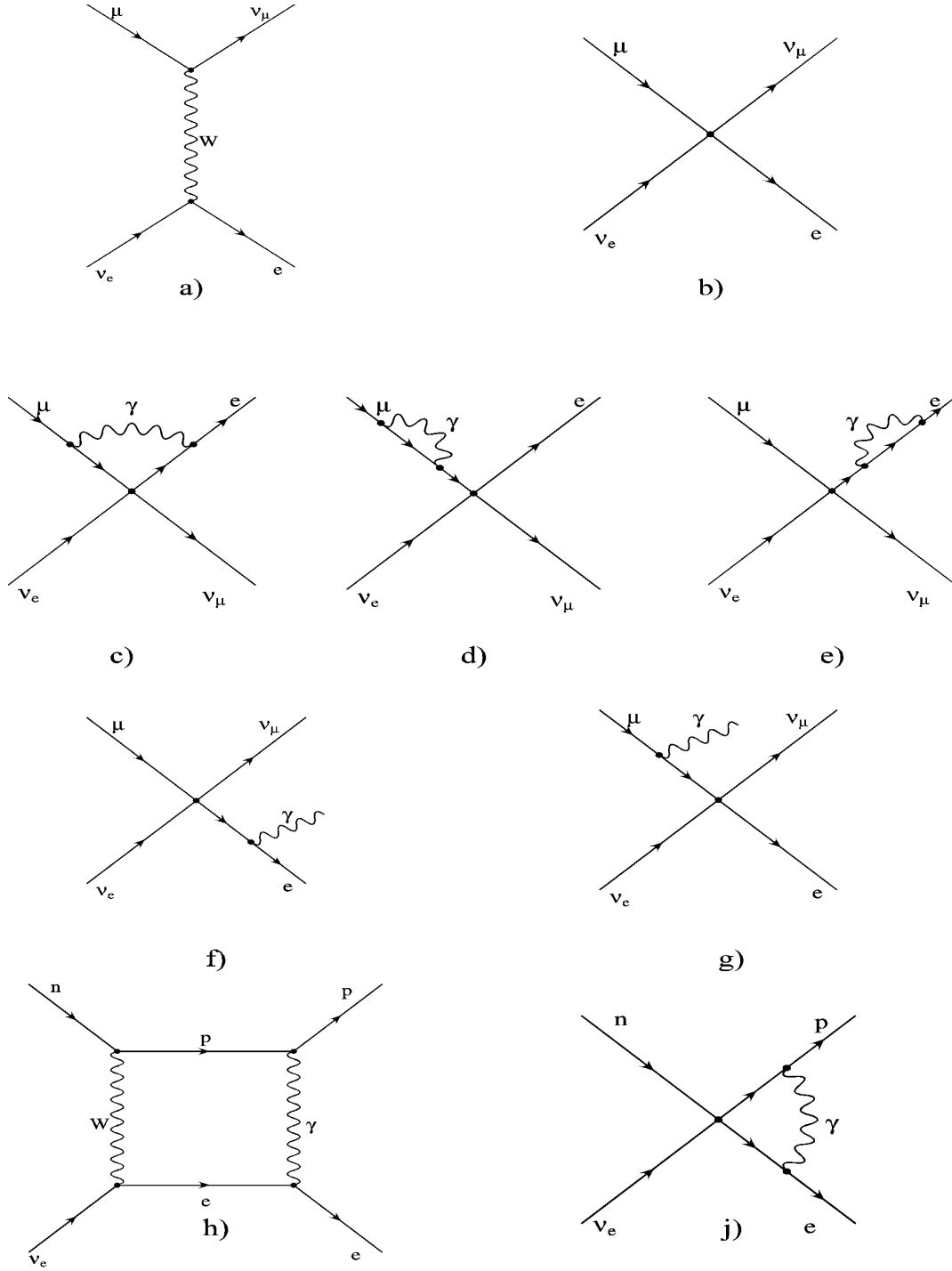


Figure 4: Muon decay in tree-diagram approximation (a) and in local four-fermion approximation (b). Electromagnetic corrections to muon decay in the local approximation (c) -(g). Electromagnetic corrections to the  $\beta$ -decay of the neutron: in tree-diagram approximation (h), in four-fermion approximation (j).

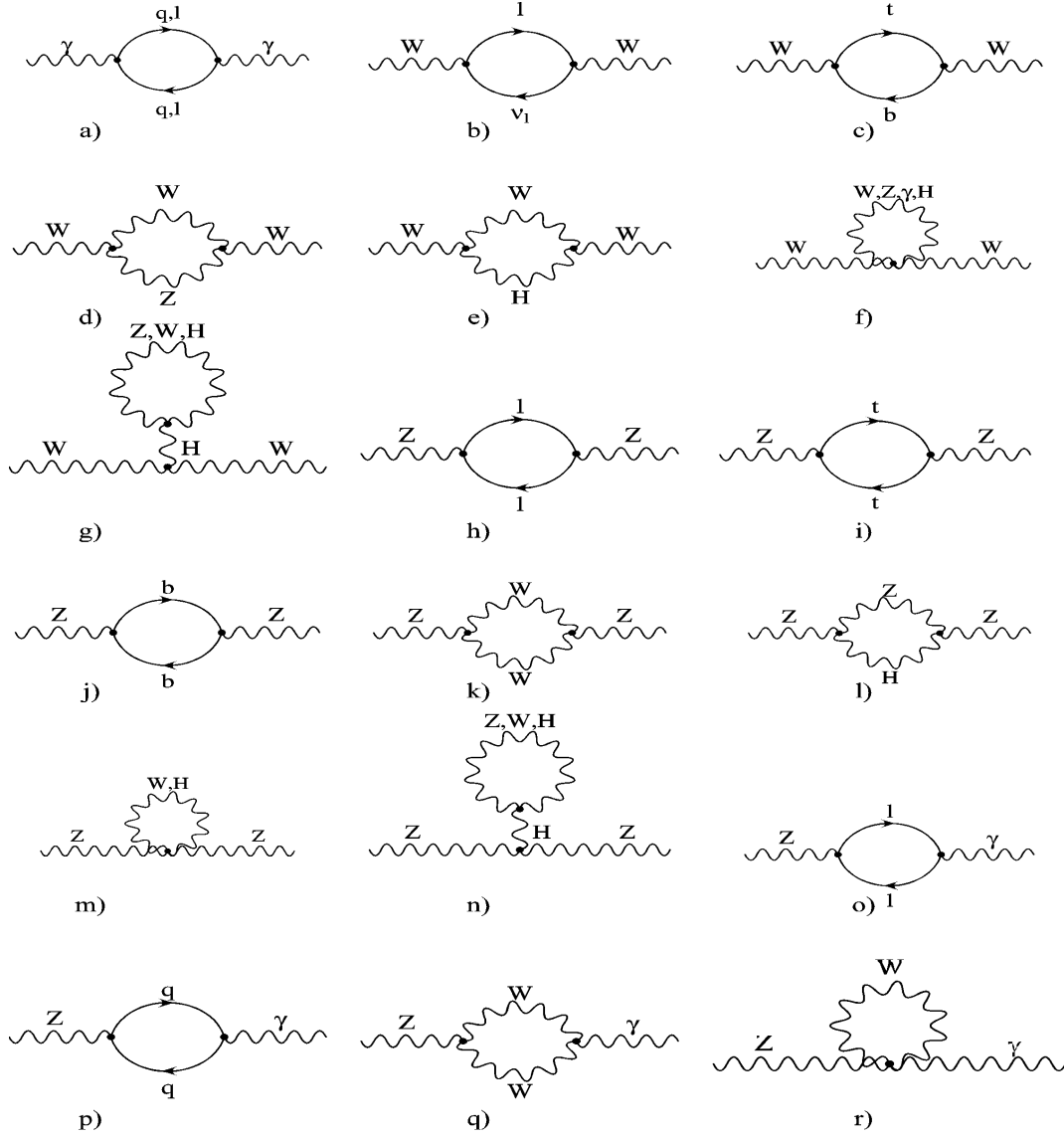


Figure 5: Photon polarization of the vacuum, resulting in the logarithmic running of the electromagnetic charge  $e$  and the ‘fine structure constant’  $\alpha \equiv \frac{e^2}{4\pi}$ , as a function of  $q^2$ , where  $q$  is the 4-momentum of the photon (a). Some of the diagram that contribute to the proper energy of the  $W$ -boson (b)-(g). Some diagrams that contribute to the proper energy of the  $Z$ -boson (h)-(n). Some of the diagram that contribute to the  $Z \leftrightarrow \gamma$  transition (o)-(r).

$G_\mu$  is expressed in terms of the fine structure constant  $\alpha$  and the masses of the  $W$  and  $Z$  bosons.

### Main relations of electroweak theory.

It is well known [4] that in the Born approximation we have (see figure 4a)

$$G_\mu = \frac{g^2}{4\sqrt{2}m_W^2} , \quad (3)$$

where  $m_W$  is the  $W$  boson mass and  $g$  is its coupling constant to the charged current.

On the other hand, we have in the same approximation

$$m_W = \frac{1}{2}g\eta , \quad (4)$$

where  $\eta$  is the vacuum expectation value of the higgs field.

Likewise,

$$m_Z = \frac{1}{2}f\eta , \quad (5)$$

where  $m_Z$  is the  $Z$  boson mass and  $f$  is its coupling constant to the neutral left-handed current.

Therefore,

$$\frac{m_W}{m_Z} = \frac{g}{f} . \quad (6)$$

If we introduce the famous Weinberg angle [1], it becomes obvious that the two definitions are equally valid in the Born approximation:

$$\begin{aligned} \cos \theta_W &= \frac{m_W}{m_Z}, \\ \cos \theta_W &= \frac{g}{f}. \end{aligned} \quad (7)$$

It is well known [1] that the angle  $\theta_W$  in the electroweak theory defines the relation between the electric charge  $e$  and the weak charge  $g$ :

$$e = g \sin \theta_W . \quad (8)$$

In the Born approximation, therefore,

$$G_\mu = \frac{g^2}{4\sqrt{2}m_W^2} = \frac{1}{\sqrt{2}\eta^2} = \frac{\pi\alpha}{\sqrt{2}m_W^2 \sin^2 \theta_W} = \frac{\pi\alpha}{\sqrt{2}m_Z^2 \sin^2 \theta_W \cos^2 \theta_W} . \quad (9)$$

If we now take into account the electroweak corrections, we find

$$\frac{m_W}{m_Z} \neq \frac{g}{f} .$$

### Traditional parametrization of corrections to the $\mu$ -decay and the running $\alpha$ .

Sirlin's definitions [23] are widely used in the literature (see the review by Langacker and Erler [35] and references therein); according to them

$$s_W^2 \equiv \sin^2 \theta_W \equiv 1 - c_W^2 \equiv 1 - \cos^2 \theta_W = 1 - \frac{m_W^2}{m_Z^2} , \quad (10)$$

$$G_\mu = \frac{\pi\alpha}{\sqrt{2}m_W^2 s_W^2 (1 - \Delta r)} , \quad (11)$$

where

$$\Delta r = \Delta r_{em} + \Delta r_{ew}$$

includes both the truly electroweak correction  $\Delta r_{ew}$  for loops in figure 3, and the purely electromagnetic correction  $\Delta r_{em}$  due to  $\alpha$  running from  $q^2 = 0$  to  $q^2 \sim m_W^2, m_Z^2$ . This correction arises because

$$\alpha \equiv \alpha(q^2 = 0) = 1/137.035985(61) \quad (12)$$

is defined for  $q^2 = 0$ , while typical momenta of virtual particles in the electroweak loop are of the order of the intermediate boson masses. It is convenient to denote

$$\bar{\alpha} \equiv \alpha(m_Z^2) = \frac{\alpha}{1 - \Delta r_{em}} \equiv \frac{\alpha}{1 - \delta\alpha} . \quad (13)$$

$\Delta r_{em}$  was calculated in a number of papers [26]; the necessary formulas are given in Appendices B and C. The contribution of the lepton loops to  $\Delta r_{em}$  is described by the expression

$$\delta\alpha^l \equiv \Delta r_{em}^l = \frac{\alpha}{3\pi} \sum_l \left[ \ln \frac{m_Z^2}{m_l^2} - \frac{5}{3} \right] = 0.03141 , \quad (14)$$

where  $l = e, \mu, \tau$ . The contribution of quark loops cannot be calculated theoretically because the quark mass in the logarithm has no rigorous theoretical definition. This reflects our ignorance of strong interactions at large distances (small momenta). The quark (hadron) part  $\Delta r_{em}^h$  is therefore calculated by substituting into the dispersion relation the experimental data on the cross section of the  $e^+e^-$ -annihilation into hadrons:

$$\delta\alpha^h \equiv \Delta r_{em}^h = \frac{m_Z^2}{4\pi^2\alpha} \int_{4m_\pi^2}^{\infty} \frac{ds}{m_Z^2 - s} \sigma_{e^+e^-}^h , \quad (15)$$

where  $\sigma_{e^+e^-}^h$  is the cross section of the  $e^+e^-$  annihilation into hadrons via one virtual photon.

In this review we make use of the recent result reported by Jegerlehner and Eidelman [26]:  $\Delta r_{em}^h = 0.02799(66)$ , so that

$$\delta\alpha = \delta\alpha^l + \delta\alpha^h \equiv \Delta r_{em} = \Delta r_{em}^l + \Delta r_{em}^h = 0.05940(66) . \quad (16)$$

As follows from (13) and (16),

$$\bar{\alpha} = [128.896(90)]^{-1} . \quad (17)$$

A summary of results of various calculations of  $\bar{\alpha}$  is given in Appendix C. Following tradition, the contributions of the  $t$ -quark loop and the  $W$ -boson loop are not included into  $\alpha(m_Z)$ . In the leading approximation in  $1/m_t^2$ , the contribution of the  $t$ -quark loop is

$$\Delta r_{em}^t = -\frac{\alpha}{\pi} \frac{4}{45} \left(\frac{m_Z}{m_t}\right)^2 = -0.00005 \quad \text{for } m_t = 180\text{GeV} \quad , \quad (18)$$

and the exact formula (see equation (67)) corresponds to

$$\Delta r_{em}^t = -0.00006 \quad \text{for } m_t = 180\text{GeV} \quad .$$

The contribution of the  $W$ -boson loop is gauge-dependent. In the 't Hooft–Feynman gauge it is  $\Delta r_{em}^W = 0.00050$  (see (66)).

### Deep inelastic neutrino scattering by nucleons.

A predominant part of theoretical work on electroweak corrections prior to the discovery of the  $W$ - and  $Z$ -bosons was devoted to calculating the neutrino–electron [27] and especially nucleon–electron [28] interaction cross sections. The reason for this is that after the discovery of neutral currents the quantity  $s_W^2 \equiv 1 - \frac{m_W^2}{m_Z^2}$  was extracted precisely from a comparison of the cross section of neutral currents (NC) and the charged currents (CC). While a  $W$ -boson interacts with a charged current of  $V - A$  type, for example,

$$\frac{g}{2\sqrt{2}} W_\alpha \bar{u}(\gamma_\alpha + \gamma_\alpha \gamma_5) d \quad , \quad (19)$$

the  $Z$ -boson interacts with neutral currents that have a more complex form,

$$\frac{f}{2} Z_\alpha \bar{\psi}_f [T_3^f \gamma_\alpha \gamma_5 + (T_3^f - 2Q^f s_W^2) \gamma_\alpha] \psi_f \quad , \quad (20)$$

where  $T_3^f$  is the third projection of the weak isotopic spin of the left-hand component of the fermion  $f$  (quark or lepton),  $Q^f$  is its charge, and  $\psi_f$  is the Dirac spinor describing it. In the tree approximation, the NC/CC cross section ratio for purely axial interactions (and isoscalar target) equals 1, since in this approximation the quantity

$$\rho = \frac{f^2}{g^2} \frac{m_W^2}{m_Z^2} \quad (21)$$

equals unity. With the vector current taken into account, the ratio of NC and CC is a function of  $s_W^2$ . Measurements of this ratio gave  $s_W^2 \approx 0.23$ , which thus made it possible to predict the masses of the  $W$  and  $Z$  bosons (from the formula for the muon decay (9)). More accurate measurements of NC/CC made it possible to improve the accuracy of  $s_W^2$  to such an extent that it became necessary to take into account the electroweak radiative corrections both in  $s_W^2$  and in  $\rho$ , which now, with the corrections taken into account, is not equal to unity any more.

Veltman was the first to point out [29] that if  $m_t/m_Z \gg 1$ , the main correction to  $\rho$  and  $s_W^2$  is caused by the violation of the electroweak isotopic invariance by the masses of

the  $t$ - and  $b$ -quarks in loops of self-energies of the  $Z$  and  $W$ -bosons. To find  $\rho$  in the limit  $m_t/m_Z \gg 1$ , it is sufficient to consider these loops neglecting the momentum of  $W$  and  $Z$  bosons  $q$  in comparison with the masses of the  $W$  and  $Z$ , i.e. for  $q^2 = 0$ . Elementary calculation of the loops indicated above yields (see Appendix H):

$$\rho = 1 + \frac{3\alpha_Z}{16\pi} \left(\frac{m_t}{m_Z}\right)^2 = 1 + \frac{3\alpha_W}{16\pi} \left(\frac{m_t}{m_W}\right)^2 = 1 + \frac{3G_\mu m_t^2}{8\sqrt{2}\pi^2} . \quad (22)$$

Here and hereafter we denote

$$\alpha_Z = \frac{f^2}{4\pi} , \quad \alpha_W = \frac{g^2}{4\pi} . \quad (23)$$

Since in real life  $m_t/m_Z \simeq 2$ , the sum of the remaining, non-leading corrections is found to be comparable to the correction proportional to  $m_t^2$  (see below).

At the present moment, the quantity  $s_W^2$  extracted from the data on deep inelastic  $\nu N$ -scattering is determined as 0.2260(48) (the global fit of the data from the collaborations CDHS [30] and CHARM [31]), or 0.2218(59) (collaboration CCFR) [32]. The accuracy of these data is poorer than found by the direct measurement of the  $W$  boson mass  $m_W$  and of the ratio  $m_W/m_Z$  by the collaborations UA2 in CERN [33] and CDF at Tevatron [34]. According to PDG [35], the fitted quantity is  $m_W = 80.22(26)$ , which corresponds to  $s_W^2 = 0.2264(25)$ . Note that according to the most recent data [36], the measurement accuracy is even higher for  $m_W$ :  $m_W = 80.26(16) \text{ GeV}$ . For this reason we will not discuss further the deep inelastic scattering of the neutrino. Moreover, additional assumptions on the effective mass of the  $c$ -quark and on the accuracy of the QCD corrections are necessary for the interpretation of these experiments.

### Other processes involving neutral currents.

This is true to even greater degree for the parity violation in eD-scattering [8], which, in addition, provides a considerably less accurate  $s_W^2 = 0.216(17)$ .

The measurement accuracy of the  $\nu_\mu e$ - and  $\bar{\nu}_\mu e$ -scattering even in the highest-accuracy experiment CHARM II [37] is not sufficient for revealing the genuine electroweak corrections. However, after an analysis in the Born approximation, this experiment has demonstrated for the first time that the interaction constant of the current  $\bar{\nu}_\mu \nu_\mu$  with the  $Z$  boson is in satisfactory agreement with the theory (see [38], [39]).

The experiment on measuring parity violation in cesium  $^{133}\text{Cs}_{55}$  [40] is also insufficiently sensitive, at the accuracy achieved. Here the effect is produced by the interaction of the nucleon vector current with the electron axial current. Since the characteristic momentum of electron in an atom is small in comparison with nuclear dimensions, all nucleons of a nucleus ‘function’ coherently and the nucleus is characterized by an aggregate weak charge  $Q_W$ , which is experimentally found to be  $Q_W^{exp} = -71.0 \pm 1.8$ , while the theoretically anticipated value is  $-72.9 \pm 0.1$ . A spectacular property of  $Q_W$  is the fact that owing to an accidental cancellation of protons’ and neutrons’ contributions, it is practically independent of  $m_t$ . On the contrary,  $Q_W$  is very sensitive to the contribution of neutral bosons  $Z'$  and  $Z''$  heavier than the  $Z$ -bosons (if they exist).

The best object for testing the electroweak theory at the loop level is therefore the  $Z$  boson with which this review is predominantly connected.

### 3 On optimal parametrization of the theory and the choice of the Born approximation.

The electroweak theory is in many ways different from electrodynamics, in particular in the diversity of particle interactions that must be taken into account when considering any effect in the loop approximation. The parametrization of QED is straightforward: fundamental quantities are the electron mass and charge, which are known with very high accuracy. It is therefore natural to express all theoretical predictions of QED in terms of  $\alpha$  and  $m_e$ .

#### Traditional choice of the main parameters.

The choice of the main parameters in electroweak theory is not equally obvious. Historically, those selected were  $G_\mu$  as an experimentally measured with best accuracy weak decay constant,  $s_W^2 \equiv 1 - m_W^2/m_Z^2$ , since  $W$  and  $Z$  bosons were not yet directly observed at that time, while the value of  $s_W^2$  was known from experiments with neutral currents, and finally,  $\alpha$ . This parametrization of the 1970s proved to be surprisingly long-lived; the loop parameters  $\Delta r$  and  $\rho$  connected with it are widely used in the literature and are very likely to survive beyond the end of this century.

In fact, this parametrization is far from being optimal because  $m_W$  (and thus  $s_W$  as well) is measured experimentally at much poorer accuracy than  $m_Z$ :

$$m_W = 80.26(16)\text{GeV} \quad (24)$$

$$m_Z = 91.1884(22)\text{GeV} \quad (25)$$

As a result,  $s_W^2$  is extracted by fitting the loop formulas for various observables. This extraction inevitably requires that we fix the values of the  $t$ -quark mass and the higgs mass. Another drawback of this parametrization is that the quantity  $\alpha$ , despite its superior accuracy, is not directly related to the electroweak loops, which are characterized by a quantity  $\bar{\alpha}$ , and these we know with much less impressive accuracy. As a result, the purely electromagnetic correction  $\Delta r_{em}$  is not separated from the genuinely electroweak corrections, thus blurring the interpretation of the experimental data.

#### Optimal choice of the main parameters.

As follows from the above remarks, the currently optimal parametrization is the one based on  $G_\mu$ ,  $m_Z$  and  $\bar{\alpha}$ . With this parametrization it is convenient to introduce the weak angle  $\theta$ , defined (by analogy to equation (9)) by the relation

$$G_\mu = \frac{\pi \bar{\alpha}}{\sqrt{2} m_Z^2 s^2 c^2} \quad , \quad (26)$$

where  $s^2 \equiv \sin^2 \theta$ ,  $c^2 \equiv \cos^2 \theta$ .

As follows from equation (26),

$$\sin^2 2\theta = \frac{4\pi \bar{\alpha}}{\sqrt{2} G_\mu m_Z^2} = 0.71078(50) \quad ,$$

$$\begin{aligned}s^2 &= 0.23110(23) \ , \\ c &= 0.87687(13) \ .\end{aligned}\tag{27}$$

The angle  $\theta$  was introduced in mid-1980s [41]. However, its consistent use began only after the publication of [42].

Using  $\theta$  instead of  $\theta_W$  automatically takes into account the running of  $\alpha$  and makes it possible to concentrate on the genuinely electroweak corrections. Using  $m_Z$  instead of  $s_W$  allows one to explicitly single out the dependence on  $m_t$  and  $m_H$  for each electroweak observable.

Note that a different definition of the  $Z$  boson mass  $\overline{m}_Z$  is known in the literature, related to a different parametrization of the shape of the  $Z$  boson peak [43]. This mass  $\overline{m}_Z$  is smaller than  $m_Z$  by approximately 30 MeV. In this review, we consistently use only  $m_Z$ , following the summary reports of LEP collaborations [44].

Let us show how the parametrization in terms of  $G_\mu$ ,  $m_Z$  and  $s$  is applied to the decay amplitudes of  $Z$  boson and to the ratio  $m_W/m_Z$ .

### **Z boson decays. Amplitudes and widths.**

In correspondence with equation (20), we rewrite the amplitude of the  $Z$  boson decay into a fermion–antifermion pair  $f\bar{f}$  in the form

$$M(Z \rightarrow f\bar{f}) = \frac{1}{2}\bar{f}Z_\alpha\bar{\psi}_f(\gamma_\alpha g_{Vf} + \gamma_\alpha\gamma_5 g_{Af})\psi_f \ .\tag{28}$$

Here by definition  $\bar{f}$  is the value of the coupling constant  $f$  in the Born approximation,

$$\bar{f}^2 = 4\sqrt{2}G_\mu m_Z^2 = 0.54866(4)\tag{29}$$

(the use of the same letter  $f$  to denote both the fermion and the coupling constant cannot lead to confusion, to such an extent are these objects different). The high accuracy with which the numerical value of  $\bar{f}$  is known comes from the fact that  $\bar{f}$  is independent of  $\bar{\alpha}$ . All electroweak radiative corrections are ‘hidden’ in dimensionless constants  $g_{Vf}$  and  $g_{Af}$ . These coefficients do not include the contribution of the final state interactions due to the exchange of gluons (for quarks) and photons (for quarks and leptons). The final state interactions have nothing in common with the electroweak corrections and must be taken into account as separate factors in the expressions for the decay rates. These factors are sometimes known as ‘radiators’, since they cover not only exchange of photons and gluons but also their emission.

Radiators are trivially unities in the case of decay to any of the neutrino pairs  $\nu_e\bar{\nu}_e$ ,  $\nu_\mu\bar{\nu}_\mu$ ,  $\nu_\tau\bar{\nu}_\tau$  and therefore

$$\Gamma_\nu = \Gamma(Z \rightarrow \nu\bar{\nu}) = 4\Gamma_0(g_{A\nu}^2 + g_{V\nu}^2) \ ,\tag{30}$$

where  $\Gamma_0$  is the so-called standard width:

$$\Gamma_0 = \frac{G_\mu m_Z^3}{24\sqrt{2}\pi} = 82.944(6)\text{MeV}.\tag{31}$$



If neutrino masses are assumed to be negligible, then

$$g_{A\nu} = g_{V\nu} \equiv g_\nu \quad (32)$$

so that

$$\Gamma_\nu = 8\Gamma_0 g_\nu^2. \quad (33)$$

For decays to any of the pairs of charged leptons  $l\bar{l}$  we have [18]

$$\Gamma_l \equiv \Gamma(Z \rightarrow l\bar{l}) = 4\Gamma_0 [g_{Vl}^2(1 + \frac{3\bar{\alpha}}{4\pi}) + g_{Al}^2(1 + \frac{3\bar{\alpha}}{4\pi} - 6\frac{m_l^2}{m_Z^2})] , \quad (34)$$

where the QED correction is taken into account only to the lowest approximation in  $\bar{\alpha}$ ; we neglect terms on the order of  $(\bar{\alpha}/\pi)^2 \sim 10^{-6}$ . The term proportional to  $m_l^2$  is negligible for  $l = e, \mu$  and must be included only for  $l = \tau$  ( $m_\tau^2/m_Z^2 = 3.8 \cdot 10^{-4}$ ).

For decays to any of the five pairs of quarks  $q\bar{q}$  we have

$$\Gamma_q \equiv \Gamma(Z \rightarrow q\bar{q}) = 12 \cdot \Gamma_0 [g_{Aq}^2 R_{Aq} + g_{Vq}^2 R_{Vq}]. \quad (35)$$

Here the factor 3, additional in comparison with leptons, takes into account the three colors of each quark. The ‘radiators’ in the first approximation are identical for the vector and the axial-vector interactions,

$$R_{Vq} = R_{Aq} = 1 + \frac{\hat{\alpha}_s}{\pi} , \quad (36)$$

where  $\hat{\alpha}_s$  is the constant of interaction of gluons with quarks at  $q^2 = m_Z^2$ . There are different conventions for the choice of  $\hat{\alpha}_s$ . In calculations of decays of the  $Z$ -boson, it is quite typical to determine  $\hat{\alpha}_s$  using the so-called modified minimal subtraction scheme,  $\overline{MS}$  (see at the end of Appendix A). The numerical value of  $\hat{\alpha}_s$ , found from  $Z$ -boson decays, is of the order of 0.12. For additional details on the value of  $\hat{\alpha}_s$  and for more accurate expressions for radiators see Appendix F. Here we only remark that vast literature is devoted to radiator calculations; they are calculated using perturbation theory up to terms  $(\hat{\alpha}_s/\pi)^3$ , see [45], [46], [47], [48]. The full hadron width is, to the accuracy of very small corrections, the sum of widths of five quark channels:

$$\Gamma_h = \Gamma_u + \Gamma_d + \Gamma_s + \Gamma_c + \Gamma_b . \quad (37)$$

The full width of the  $Z$  boson is given by the obvious expression:

$$\Gamma_Z = \Gamma_h + \Gamma_e + \Gamma_\mu + \Gamma_\tau + 3\Gamma_\nu . \quad (38)$$

The annihilation cross section  $e^+e^-$  into hadrons at the  $Z$ -peak is given by the Breit–Wigner formula:

$$\sigma_h = \frac{12\pi}{M_Z^2} \frac{\Gamma_e \Gamma_h}{\Gamma_Z^2} . \quad (39)$$

Finally, the following notation for the ratio of partial widths is widely used:

$$R_b = \frac{\Gamma_b}{\Gamma_h} , \quad R_c = \frac{\Gamma_c}{\Gamma_h} , \quad R_l = \frac{\Gamma_h}{\Gamma_l} . \quad (40)$$

(Note that in contrast to  $R_b$  and  $R_c$ ,  $\Gamma_h$  in  $R_l$  is in the numerator.)

## Asymmetries.

In addition to the total and partial widths of  $Z$  boson decays, experimentalists also measure effects due to parity non-conservation, i.e. the interference of the vector and axial-vector currents. For pairs of light quarks ( $u, d, s, c$ ) and leptons we determine the quantity

$$A_f = \frac{2g_{Af}g_{Vf}}{g_{Af}^2 + g_{Vf}^2} . \quad (41)$$

For the pair  $b\bar{b}$  (see Appendix G),

$$A_b = \frac{2g_{Ab}g_{Vb}}{v^2g_{Ab}^2 + \frac{1}{2}(3-v^2)g_{Vb}^2} \cdot v , \quad (42)$$

where  $v$  is the velocity of the  $b$ -quark (in units of  $c$ ):

$$v = \sqrt{1 - \frac{4\hat{m}_b^2}{m_Z^2}} . \quad (43)$$

Here  $\hat{m}_b$  is the value of the ‘running mass’ of the  $b$ -quark with momentum  $m_Z$ , calculated in the scheme  $\overline{MS}$  [50]. The forward-backward charge asymmetry in the decay to  $f\bar{f}$  equals (see Appendix G)

$$A_{FB}^f \equiv \frac{N_F - N_B}{N_F + N_B} = \frac{3}{4}A_e A_f , \quad (44)$$

where  $A_e$  refers to the creation of a  $Z$  boson in  $e^+e^-$ -annihilations, and  $A_f$  refers to its decay into  $f\bar{f}$ .

The longitudinal polarization of the  $\tau$ -lepton in the  $Z \rightarrow \tau\bar{\tau}$  decay is

$$P_\tau = -A_\tau . \quad (45)$$

If, however, we measure polarization as a function of the angle  $\theta$  between the momentum of a  $\tau^-$  and the direction of the electron beam, this permits the determination of not only  $A_\tau$  but  $A_e$  as well:

$$P_\tau(\cos\theta) = -\frac{A_\tau(1 + \cos^2\theta) + A_e 2\cos\theta}{(1 + \cos^2\theta) + A_\tau A_e 2\cos\theta} . \quad (46)$$

The polarization of  $P_\tau$  is found from  $P_\tau(\cos\theta)$  by separately integrating the numerator and the denominator in (46) over the total solid angle.

The relative difference between total cross sections in the  $Z$  peak for the left- and right-polarized electron that collide with non-polarized positrons (this quantity is measured at the SLC collider) is

$$A_{LR} \equiv \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} = A_e . \quad (47)$$

The measurement of the asymmetries outlined above allows one to experimentally determine the quantities  $g_{Vf}/g_{Af}$ , since these asymmetries are caused by the interference of the vector and axial-vector currents. In their turn, measurements of the widths  $\Gamma_f$ ,  $\Gamma_h$ ,  $\Gamma_Z$  mostly permit the experimental determination of  $g_{Af}$ , since  $|g_{Vq}|^2 < |g_{Aq}|^2$  for quarks, and for leptons  $|g_{Vl}|^2 \ll |g_{Al}|^2$ . As for  $\Gamma_q$  and  $\Gamma_h$ , getting these quantities allows one to find  $\hat{\alpha}_s$ .

## The Born approximation for hadronless observables.

Before discussing the loop electroweak corrections, let us consider expressions for  $m_W/m_Z$ ,  $g_{Af}$  and  $g_{Vf}/g_{Af}$  in the so-called  $\bar{\alpha}$ -Born approximation. Using the angle  $\theta$  introduced earlier, its  $\sin \theta \equiv s$  and  $\cos \theta \equiv c$ , we automatically take into account the purely electromagnetic correction due to the running of  $\alpha$ . It is easily shown that in the  $\bar{\alpha}$ -Born approximation

$$(m_W/m_Z)^B = c \quad (48)$$

$$g_{Af}^B = T_{3f} \quad (49)$$

$$(g_{Vf}/g_{Af})^B = 1 - 4|Q_f|s^2 \quad (50)$$

It is of interest to compare the Born values with their experimental values. Table 1 presents this comparison for the so-called ‘hadronless’ observables (here and below the experimental results are taken from [44].)

**Table 1**

Observable	Experiment	$\bar{\alpha}$ -Born
$m_W/m_Z$	0.8802(18)	0.8769(1)
$m_W$ (GeV)	80.26(16)	79.96(2)
$s_W^2$	0.2253(31)	0.2311(2)
$g_{Al}$	-0.5011(4)	-0.5000(0)
$\Gamma_l$ (MeV)	83.93(14)	83.57(2)
$g_{Vl}/g_{Al}$	0.0756(14)	0.0756(9)
$s_l^2$	0.2311(4)	0.2311(2)

For the reader’s convenience, the table lists different representations of the same observables known in the literature. Thus, according to widely used definitions,

$$s_W^2 = 1 - m_W^2/m_Z^2 \quad , \quad (51)$$

$$s_l^2 \equiv s_{eff}^2 \equiv \sin^2 \theta_{eff}^{lept} \equiv \frac{1}{4}(1 - g_{Vl}/g_{Al}) \quad . \quad (52)$$

The experimental value of  $s_l^2$  in the table is the average of two numbers, 0.2316(5) (LEP) and 0.2305(5) (SLC). It is assumed in the table that the lepton universality holds, thus the lepton decay data have been averaged over a number of observables.

Table 1 shows that the  $\bar{\alpha}$ -Born approximation provides good description of the experimental data. (An agreement is found for the hadron decays of  $Z$ -bosons as well). An especially (and unexpectedly!) good agreement, and the ensuing smallness of radiative corrections, is found for  $g_{Vl}/g_{Al}$ . The anomalous smallness of true electroweak corrections was first pointed out in 1992 [42], when the  $\bar{\alpha}$ -Born approximation was applied for the first time. (Before that it was hidden in the shadow of the large contribution of purely electromagnetic running of  $\alpha$ , that was not separated from truly electroweak corrections). The LEP1 data of 1992 were not sufficiently accurate to allow detecting them. Even the data presented to

the Marseille conference in summer 1993 were not, as pointed out in a number of reports [51], sufficiently accurate for the detection of radiative corrections. At the Glasgow conference in summer 1994 corrections were detectable at the level of  $2.3\sigma$  for  $g_{Al}$ ,  $1.5\sigma$  for  $m_W$  and  $1\sigma$  for  $g_{Vl}/g_{Al}$  [52]. Note that the difference between most recent experimental values  $s_W^2 = 0.2253(31)$  and  $s_l^2 = 0.2311(4)$  is a  $2\sigma$  manifestation of electroweak radiative corrections, which is independent of either the choice of the Born approximation or of the choice of calculation scheme.

## 4 One-loop corrections to hadronless observables.

The fact that the experimentally observed electroweak corrections are small suggests that one-loop approximation would be sufficient for describing them [42]. This idea is supported by a thorough evaluation [18] of theoretical uncertainties contributed by higher-order perturbation theory in electroweak interaction. There exists essentially a single two-loop diagram that should be taken into account. It was calculated in [53], and we will discuss its contribution and take it into account below (see formula (79)).

### Four types of Feynman diagrams.

Four types of Feynman diagrams contribute to electroweak corrections for the observables of interest to us here,  $m_W/m_Z$ ,  $g_{Al}$ ,  $g_{Vl}/g_{Al}$ :

1. Self-energy loops for  $W$  and  $Z$  bosons with virtual  $\nu, l, q, H, W$  and  $Z$  in loops. Examples of some of these diagrams are shown in figures 5b–5n.
2. Loops of charged particles that result in  $Z \leftrightarrow \gamma$  transitions (figures 5o–5r).
3. Vertex triangles with virtual leptons and a  $W$  or  $Z$  boson (figures 6a–6c).
4. Electroweak corrections to lepton wavefunctions (figures 6d–6e).

It must be emphasized that loops shown in figures 5h–5n contribute not only to the  $m_Z$  mass and, consequently, to the  $m_W/m_Z$  ratio but also to the  $Z$  boson decay to  $l\bar{l}$ , to which  $Z \leftrightarrow \gamma$  transitions also contribute (figures 5o–5r). This occurs owing to the diagrams of the type of figures 6f–6g which give corrections to the  $Z$ -boson wavefunction.

Obviously, electroweak corrections to  $m_W/m_Z$ ,  $g_{Al}$  and  $g_{Vl}/g_{Al}$  are dimensionless and thus can be expressed in terms of  $\bar{\alpha}$ ,  $c$ ,  $s$  and the dimensionless parameters

$$t = \left(\frac{m_t}{m_Z}\right)^2, \quad h = \left(\frac{m_H}{m_Z}\right)^2,$$

where  $m_t$  is the mass of the  $t$ -quark and  $m_H$  is the higgs mass. (We neglect the masses of leptons and all quarks except  $t$ .)

**The asymptotic limit at  $m_t^2 \gg m_Z^2$ .**  
**The functions  $V_m(t, h)$ ,  $V_A(t, h)$  and  $V_R(t, h)$ .**

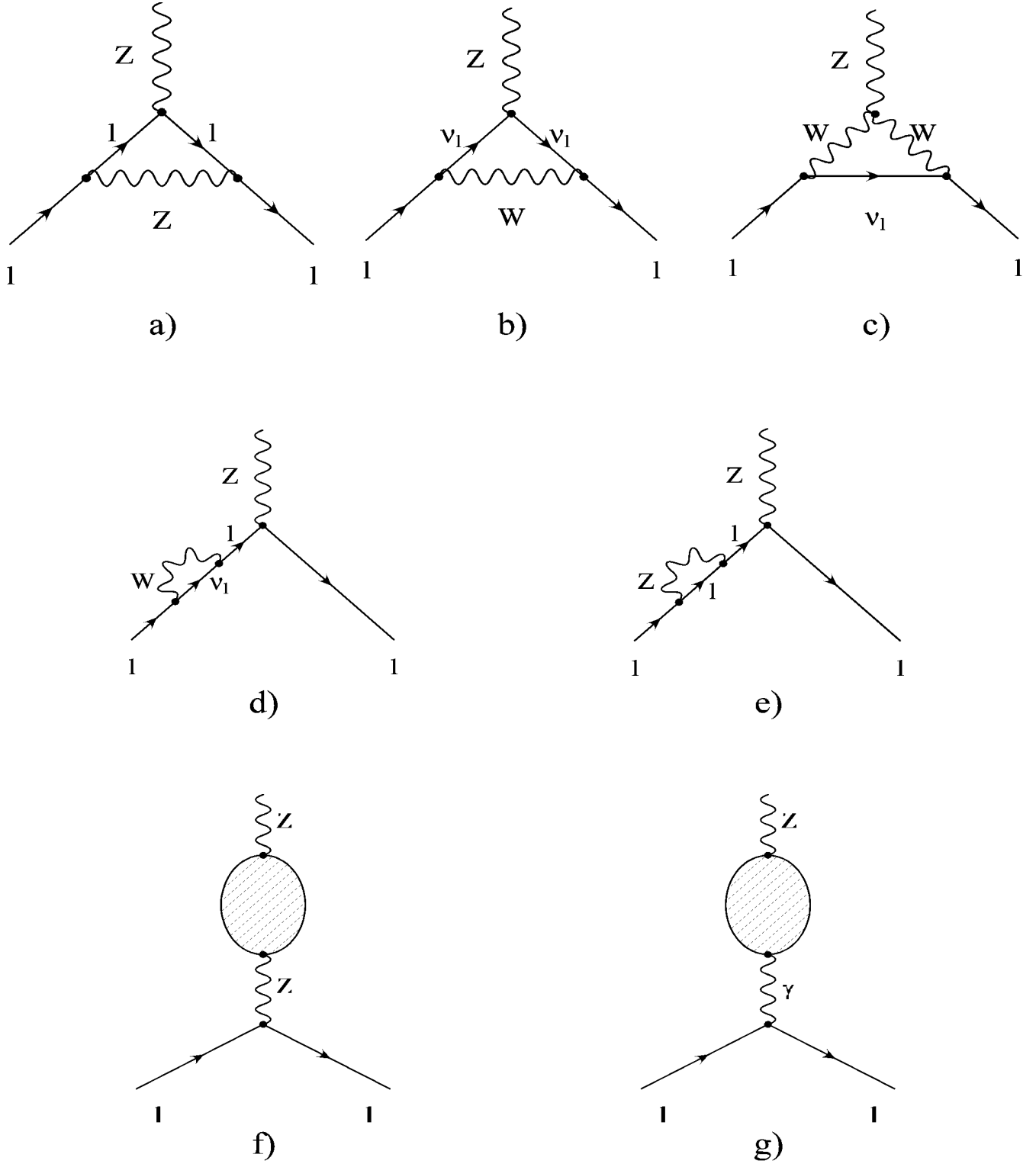


Figure 6: Vertex triangular diagrams in the  $Z \rightarrow l\bar{l}$  decay (a), (b), (c). Loops that renormalize the lepton wavefunctions in the  $Z \rightarrow l\bar{l}$  decay. (Of course, antilepton have similar loops.) (d), (e). Types of diagrams that renormalize the  $Z$  boson wavefunction in the  $Z \rightarrow l\bar{l}$  decay (f), (g). Virtual particles in the loops are those that we were discussing above

It is convenient to split the calculation of corrections into a number of stages and begin with calculating the asymptotic limit for  $t \gg 1$ .

According to the reasons mentioned above (see equation (22), the main contribution comes from diagrams that contain  $t$ - and  $b$ -quarks (5c and 5i,j). A simple calculation (see Appendix H) gives the following result for the sum of the Born and loop terms:

$$m_W/m_Z = c + \frac{3c}{32\pi s^2(c^2 - s^2)}\bar{\alpha}t \quad . \quad (53)$$

$$g_{Al} = -\frac{1}{2} - \frac{3}{64\pi s^2 c^2}\bar{\alpha}t \quad , \quad (54)$$

$$R \equiv g_{Vl}/g_{Al} = 1 - 4s^2 + \frac{3}{4\pi(c^2 - s^2)}\bar{\alpha}t \quad , \quad (55)$$

$$g_\nu = \frac{1}{2} + \frac{3}{64\pi s^2 c^2}\bar{\alpha}t \quad . \quad (56)$$

**The functions  $V_m(t, h)$ ,  $V_A(t, h)$  and  $V_R(t, h)$ .**

If we now switch from the asymptotic case of  $t \gg 1$  to the case of  $t \sim 1$ , then, first, the change in the contribution of the diagrams 5c, 5i and 5j can be written in the form

$$t \rightarrow t + T_i(t) \quad , \quad (57)$$

where the index  $i = m, A, R, \nu$  for  $m_W/m_Z$ ,  $g_{Al}$ ,  $R \equiv g_{Vl}/g_{Al}$  and  $g_\nu$ , respectively.

The functions  $T_i$  are relatively simple combinations of algebraic and logarithmic functions. They are listed in explicit form in Appendix I. Their numerical values for a range of values of  $m_t$  are given in Table 2. The functions  $T_i(t)$  thus describe the contribution of the quark doublet  $t, b$  to  $m_W/m_Z$ ,  $g_A$ ,  $R = g_{Vl}/g_{Al}$  and  $g_\nu$ . If, however, we now take into account the contributions of the remaining virtual particles, then the result can be given in the form

$$t \rightarrow V_i(t, h) = t + T_i(t) + H_i(h) + C_i + \delta V_i(t, h) \quad . \quad (58)$$

**Table 2**

$m_t$ (GeV)	$t$	$T_m$	$T_A$	$T_R$
0	0	-0.188	0.875	0.444
10	0.012	0.192	0.934	0.038
20	0.048	-0.256	0.955	-0.015
30	0.108	-0.430	0.812	-0.305
40	0.192	-0.753	0.403	-0.959
50	0.301	-0.985	0.111	-0.748
60	0.433	-0.931	0.327	-0.412
70	0.589	-0.688	0.390	-0.250
80	0.770	-0.317	0.421	-0.143
90	0.974	-0.080	0.440	-0.061
100	1.203	0.084	0.451	0.006
110	1.455	0.214	0.460	0.062
120	1.732	0.323	0.465	0.111
130	2.032	0.418	0.470	0.154
140	2.357	0.503	0.473	0.193
150	2.706	0.579	0.476	0.228
160	3.079	0.649	0.478	0.261
170	3.476	0.713	0.480	0.291
180	3.896	0.772	0.481	0.319
190	4.341	0.828	0.483	0.345
200	4.810	0.880	0.484	0.370
210	5.303	0.929	0.485	0.393
220	5.821	0.975	0.485	0.415
230	6.362	1.019	0.486	0.436
240	6.927	1.061	0.487	0.456
250	7.516	1.101	0.487	0.475
260	8.130	1.139	0.487	0.493
270	8.767	1.176	0.488	0.510
280	9.428	1.211	0.488	0.527
290	10.114	1.245	0.489	0.543
300	10.823	1.277	0.489	0.559

Here  $H_i(h)$  contain the contribution of the virtual vector and higgs bosons  $W, Z$  and  $H$  and are functions of the higgs mass  $m_H$ . (The masses of the  $W$  and  $Z$  bosons enter  $H_i(h)$  via the parameters  $c, s$ , defined by equation (26)). The explicit form of the functions  $H_i$  is given in Appendix I, and their numerical values for various values of  $m_H$  are given in Table 3.

**Table 3**

$m_H$ (GeV)	$h$	$H_m$	$H_A$	$H_R$
0.01	0.000	1.120	-8.716	1.359
0.10	0.000	1.119	-5.654	1.354
1.00	0.000	1.103	-2.652	1.315
10.00	0.012	0.980	-0.133	1.016
50.00	0.301	0.661	0.645	0.360
100.00	1.203	0.433	0.653	-0.022
150.00	2.706	0.275	0.588	-0.258
200.00	4.810	0.151	0.518	-0.430
250.00	7.516	0.050	0.452	-0.566
300.00	10.823	-0.037	0.392	-0.679
350.00	14.732	-0.112	0.338	-0.776
400.00	19.241	-0.178	0.289	-0.860
450.00	24.352	-0.238	0.244	-0.936
500.00	30.065	-0.292	0.202	-1.004
550.00	36.378	-0.341	0.164	-1.065
600.00	43.293	-0.387	0.128	-1.122
650.00	50.809	-0.429	0.095	-1.175
700.00	58.927	-0.469	0.064	-1.223
750.00	67.646	-0.506	0.035	-1.269
800.00	76.966	-0.540	0.007	-1.311
850.00	86.887	-0.573	-0.019	-1.352
900.00	97.410	-0.604	-0.044	-1.390
950.00	108.534	-0.633	-0.067	-1.426
1000.00	120.259	-0.661	-0.090	-1.460

The constants  $C_i$  in (58) include the contributions of light fermions to the self-energy of the  $W$  and  $Z$  bosons, and also to the diagrams of figure 3, describing the muon decay, and of figures 6a–6c, describing the  $Z$ -boson decay. The constants  $C_i$  are relatively complicated functions of  $s^2$  (see Appendix M). We list here their numerical values for  $s^2 = 0.23110 - \delta s^2$ :

$$C_m = -1.3497 + 4.13\delta s^2 \quad , \quad (59)$$

$$C_A = -2.2621 - 2.63\delta s^2 \quad , \quad (60)$$

$$C_R = -3.5045 - 5.72\delta s^2 \quad , \quad (61)$$

$$C_\nu = -1.1641 - 4.88\delta s^2 \quad . \quad (62)$$

**Corrections  $\delta \mathbf{V}_i(\mathbf{t}, \mathbf{h})$ .**

Finally, the last term in equation (58) includes the sum of corrections of five different types. Their common feature is that they are all quite small (except for  $\delta_2^t V_i$ ) and that they represent two loops (with the exception of a one-loop  $\delta_1 V_i$  and a three-loop  $\delta_3 V_i$ ).



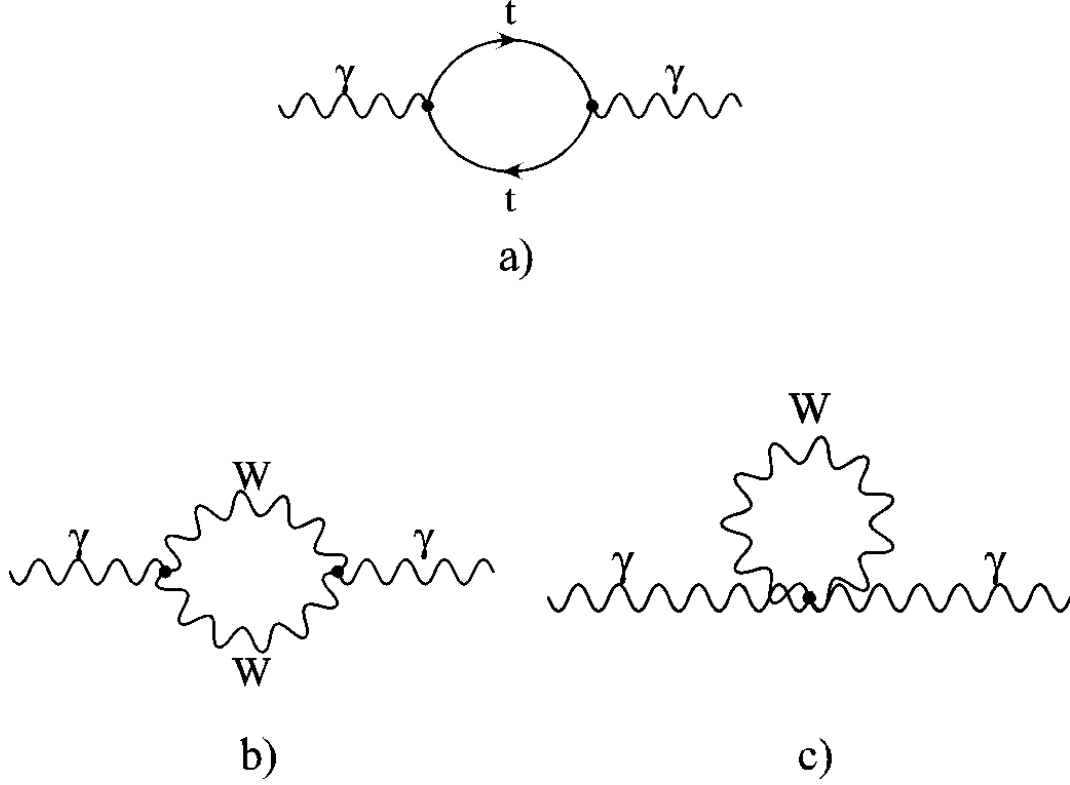


Figure 7: Virtual  $t$ -quarks (a) and  $W$  bosons (b), (c) in the photon polarization of the vacuum.

1.  $\delta_1 V_i$  contains contributions of the  $W$ -boson and the  $t$ -quark to the polarization of the electromagnetic vacuum  $\delta_W \alpha \equiv \Delta r_{em}^W$  and  $\delta_t \alpha \equiv \Delta r_{em}^t$  (see figures 7a-c and equation (16)), which traditionally are not included into the running of  $\alpha(q^2)$ , i.e. into  $\bar{\alpha}$ . It is reasonable to treat them as electroweak corrections. This is especially true for the  $W$ -loop that depends on the gauge chosen for the description of the  $W$  and  $Z$  bosons. Only after this loop is taken into account, the resultant electroweak corrections become gauge-invariant, as it should indeed be for physical observables. Here and hereafter in the calculations the 't Hooft–Feynman gauge is used (see Appendix A),

$$\delta_1 V_m(t, h) = -\frac{16}{3}\pi s^4 \frac{1}{\alpha} (\delta_W \alpha + \delta_t \alpha) = -0.055 \quad , \quad (63)$$

$$\delta_1 V_R(t, h) = -\frac{16}{3}\pi s^2 c^2 \frac{1}{\alpha} (\delta_W \alpha + \delta_t \alpha) = -0.181 \quad , \quad (64)$$

$$\delta_1 V_A(t, h) = \delta_1 V_\nu(t, h) = 0 \quad , \quad (65)$$

where

$$\frac{\delta_W \alpha}{\alpha} = \frac{1}{2\pi} [(3 + 4c^2)(1 - \sqrt{4c^2 - 1} \arcsin \frac{1}{2c}) - \frac{1}{3}] = 0.0686 \quad , \quad (66)$$

$$\frac{\delta_t \alpha}{\alpha} = -\frac{4}{9\pi} [(1 + 2t)F_t(t) - \frac{1}{3}] \simeq -\frac{4}{45\pi} \frac{1}{t} + \dots \simeq -0.00768 \quad . \quad (67)$$

(Here and in what follows, unless specified otherwise, we use  $m_t = 175$  GeV in numerical evaluations.)

2. The corrections  $\delta_2 V_i$  are caused by including virtual gluons in electroweak loops in the order  $\bar{\alpha}\hat{\alpha}_s$  (see figures 8a–c); similar diagrams can, of course, be drawn for  $W$ -bosons. In addition to loops with light quarks  $q = u, d, s, c$ , there exist similar loops with third-generation quarks  $t$  and  $b$ :

$$\delta_2 V_i(t) = \delta_2^q V_i + \delta_2^t V_i(t) \quad .$$

The analytical expressions for corrections  $\delta_2^q V_i$  and  $\delta_2^t V_i(t)$  are given in Appendix O. Here we only give numerical estimates for them,

$$\delta_2^q V_m = -0.377 \frac{\hat{\alpha}_s}{\pi} \quad , \quad (68)$$

$$\delta_2^q V_A = 1.750 \frac{\hat{\alpha}_s}{\pi} \quad , \quad (69)$$

$$\delta_2^q V_R = 0 \quad , \quad (70)$$

$$\delta_2^t V_m(t) = -11.67 \frac{\hat{\alpha}_s(m_t)}{\pi} = -10.61 \frac{\hat{\alpha}_s}{\pi} \quad , \quad (71)$$

$$\delta_2^t V_A(t) = -10.10 \frac{\hat{\alpha}_s(m_t)}{\pi} = -9.18 \frac{\hat{\alpha}_s}{\pi} \quad , \quad (72)$$

$$\delta_2^t V_R(t) = -11.88 \frac{\hat{\alpha}_s(m_t)}{\pi} = -10.80 \frac{\hat{\alpha}_s}{\pi} \quad , \quad (73)$$

since [16]

$$\hat{\alpha}_s(m_t) = \frac{\hat{\alpha}_s}{1 + \frac{23}{12\pi} \hat{\alpha}_s \ln t} \quad . \quad (74)$$

(For numerical evaluation, we use  $\hat{\alpha}_s \equiv \hat{\alpha}_s(m_Z) = 0.125$ .) We have mentioned already that the corrections  $\delta_2^t V_i(t)$ , whose numerical values were given in (71)–(73), are much larger than all other terms included in  $\delta V_i$ . We emphasize that the term in  $\delta_2^t V_i$  that is leading for high  $t$  is universal: it is independent of  $i$ . As shown in [54], this leading term is obtained by multiplying the Veltman asymptotics  $t$  by a factor

$$1 - \frac{2\pi^2 + 6}{9} \frac{\hat{\alpha}_s(m_t)}{\pi} \quad , \quad (75)$$

or, numerically,

$$t \rightarrow t \left(1 - 2.86 \frac{\hat{\alpha}_s(m_t)}{\pi}\right) \quad . \quad (76)$$

Qualitatively the factor (75) corresponds to the fact that the running mass of the  $t$ -quark at momenta  $p^2 \sim m_t^2$  that circulate in the  $t$ -quark loop is lower than ”on the mass-shell” mass of the  $t$ -quark. It is interesting to compare the correction (76) with the quantity

$$\tilde{m}_t^2 \equiv m_t^2(p_t^2 = -m_t^2) = m_t^2 \left(1 - 2.78 \frac{\hat{\alpha}_s(m_t)}{\pi}\right) \quad , \quad (77)$$

calculated in the Landau gauge in [55], p 102. The agreement is overwhelming. There is, therefore, a simple mnemonic rule for evaluating the main gluon corrections for the  $t$ -loop.

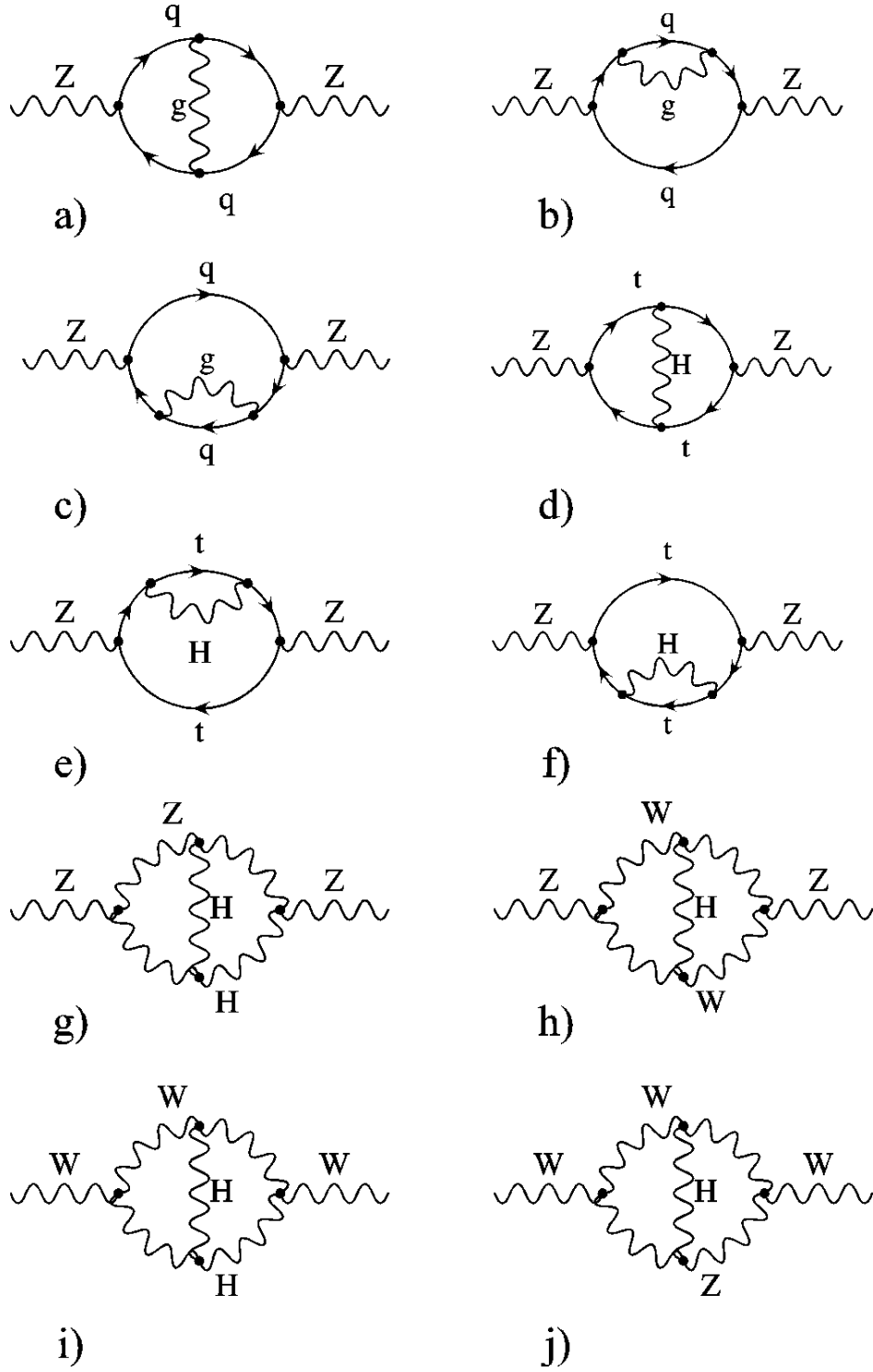


Figure 8: Gluon corrections to the electroweak quark loop of the proper energy of the  $Z$  boson (a)-(c). Higgs corrections to the electroweak  $t$  quark loop of the  $Z$  boson (d)-(f). Two-loop higgs corrections (g)-(j).

3. Corrections  $\delta_3 V_i$  of the order of  $\bar{\alpha} \hat{\alpha}_s^2$  were calculated in the literature [56] for the term leading in  $t$  (i.e.  $\bar{\alpha} \hat{\alpha}_s^2 t$ ). They are independent of  $i$  (in numerical estimates we use for the number of quark flavors  $N_f = 5$ ):

$$\delta_3 V_i(t) \simeq -(2.38 - 0.18 N_f) \hat{\alpha}_s^2(m_t) t \simeq -1.48 \hat{\alpha}_s^2(m_t) t = -0.07 \quad . \quad (78)$$

The corrections  $\delta_1 V_i, \delta_2 V_i, \delta_3 V_i$ , are independent of  $m_H$ , the corrections  $\delta_4 V_i$  depend both on  $m_t$ , and on  $m_H$ , while the corrections  $\delta_5 V_i$  are proportional to  $m_H^2$ . In contrast to all previous corrections, they arise due to the electroweak interaction in two loops, not one.

4. In the leading approximation in  $t$  the correction  $\delta_4 V_i(t, h)$  produced by the diagrams of figures 8d-f is independent of  $i$  and takes the form

$$\delta_4 V_i(t, h) = -\frac{\bar{\alpha}}{16\pi s^2 c^2} A\left(\frac{h}{t}\right) t^2 \quad , \quad (79)$$

where the function  $A(h/t)$ , calculated in [53], is given in Table 4 for  $m_H/m_t < 4$ .

For  $m_t = 175 \text{ GeV}$  and  $m_H = 300 \text{ GeV}$

$$\delta_4 V_i = -0.11 \quad . \quad (80)$$

The following expansion holds for  $m_H/m_t > 4$ :

$$\begin{aligned} A(h/t) = & -\frac{49}{4} - \pi^2 - \frac{27}{2} \ln r - \frac{3}{2} \ln^2 r - \frac{1}{3} r (2 - 12\pi^2 + 12 \ln r - 27 \ln^2 r) \\ & - \frac{r^2}{48} (1613 - 240\pi^2 - 1500 \ln r - 720 \ln^2 r) \quad , \end{aligned} \quad (81)$$

where  $r = t/h$ .  $\delta_4 V_i(t, h)$  is the greatest of the two-loop corrections in electroweak interaction; however, it is also several times smaller than the main gluon corrections  $\delta_2^t V_i$ .

Table 4

$m_H/m_t$	$A(m_H/m_t)$	$\tau^{(2)}(m_H/m_t)$	$m_H/m_t$	$A(m_H/m_t)$	$\tau^{(2)}(m_H/m_t)$
0.00	0.739	5.710	2.10	9.655	1.373
0.10	1.821	4.671	2.20	9.815	1.421
0.20	2.704	3.901	2.30	9.964	1.475
0.30	3.462	3.304	2.40	10.104	1.533
0.40	4.127	2.834	2.50	10.235	1.595
0.50	4.720	2.461	2.60	10.358	1.661
0.60	5.254	2.163	2.70	10.473	1.730
0.70	5.737	1.924	2.80	10.581	1.801
0.80	6.179	1.735	2.90	10.683	1.875
0.90	6.583	1.586	3.00	10.777	1.951
1.00	6.956	1.470	3.10	10.866	2.029
1.10	7.299	1.382	3.20	10.949	2.109
1.20	7.617	1.317	3.30	11.026	2.190
1.30	7.912	1.272	3.40	11.098	2.272
1.40	8.186	1.245	3.50	11.165	2.356
1.50	8.441	1.232	3.60	11.228	2.441
1.60	8.679	1.232	3.70	11.286	2.526
1.70	8.902	1.243	3.80	11.340	2.613
1.80	9.109	1.264	3.90	11.390	2.700
1.90	9.303	1.293	4.00	11.436	2.788
2.00	9.485	1.330			

5. Corrections  $\delta_5 V_i$  due to two-loop diagrams of the type of figure 8g-8j. They are negligible, but for the sake of completeness of the presentation, we list them in Appendix P.

### Accidental (?) compensation and the mass of the t-quark.

Now that we have expressions for all terms in formula (58), it will be convenient to analyze their roles and the general behavior of the functions  $V_i(t, h)$ . As functions of  $m_t$  at three fixed values of  $m_H$ , they are shown in figures 9a, 10a, 11a. On all these figures, we see a cusp at  $m_t = m_Z/2$ . This is a typical threshold singularity that arises when the channel  $Z \rightarrow t\bar{t}$  is opened. It is of no practical significance since experiments give  $m_t \gg m_Z/2$ . What really impresses is that the functions  $V_i$  are nearly zero in the interval  $m_t \sim 100 - 200$  GeV. This happens because of the compensation of the leading term  $t$  and the rest of the terms which produce a negative aggregate contribution. This is especially well pronounced in the function  $V_R$  at  $m_t \sim 180$  GeV. Here the main negative contribution comes from the light fermions (the constant  $C_R$ ).

If we neglect the small correction  $\delta_4 V_i(t, h)$  which depends both on  $t$  and on  $h$ , then each function  $V_i(t, h)$  is a sum of functions one of which is  $t$ -dependent but independent of  $h$ , while the second is  $h$ -dependent but independent of  $t$  (plus, of course, a constant which is

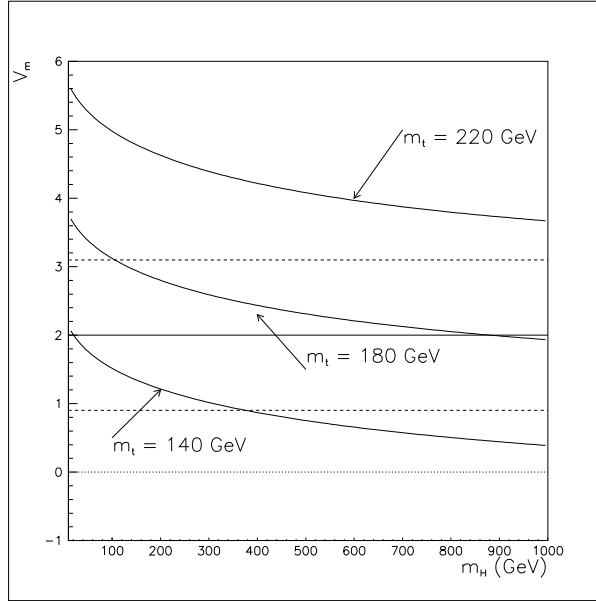
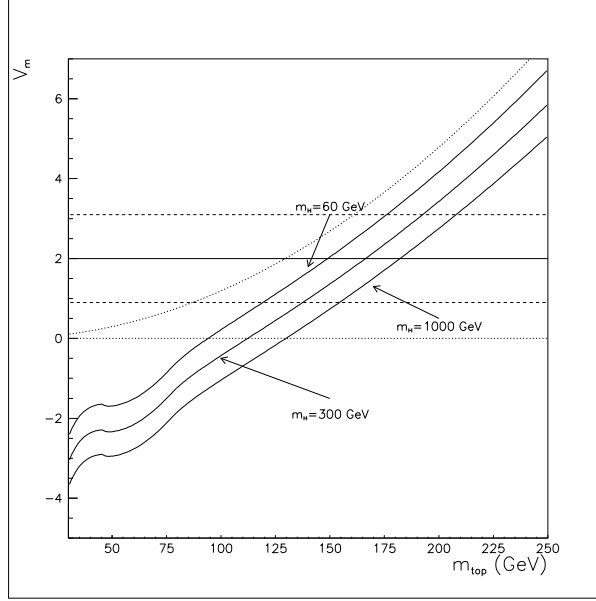


Figure 9:  $V_m$  as a function of  $m_t$  for three values of  $m_H$ : 60, 300 and 1000 GeV, according to equation (58). The dashed-curve parabola  $t = (m_t/m_Z)^2$  corresponds to the Veltman approximation. Solid horizontal line traces the experimental value of  $V_m^{\text{exp}}$  while the long-dash horizontal lines give its upper and lower limits of the  $1\sigma$  level (a).  $V_m$  as a function of  $m_H$  for three values of  $m_t$ : 150, 175 and 200 GeV, according to equation (58). (Horizontal lines the same as in figure 9(a). )

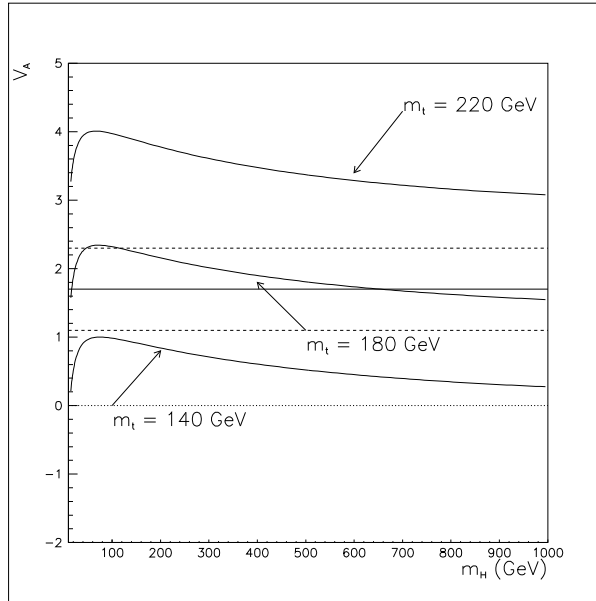
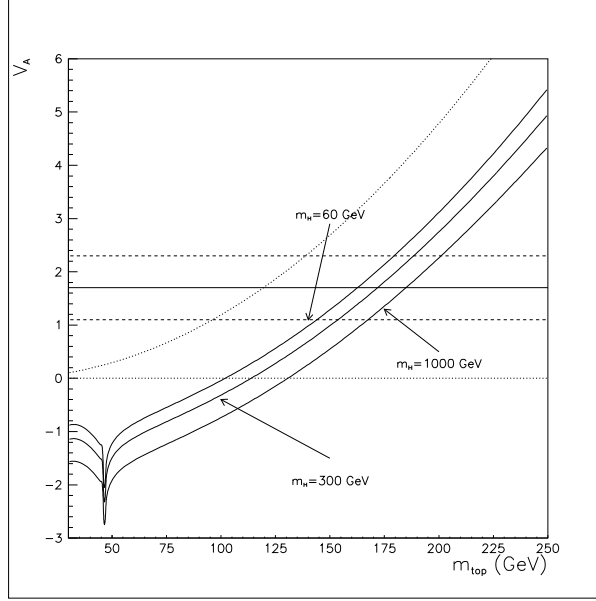


Figure 10:  $V_A$  as a function of  $m_t$ . (The remaining clarifications are similar to those given to figure 9) (a).  $V_A$  as a function  $m_H$  for three values of  $m_t$ : 150, 175 and 200 GeV. (See caption to figure 9) (b)

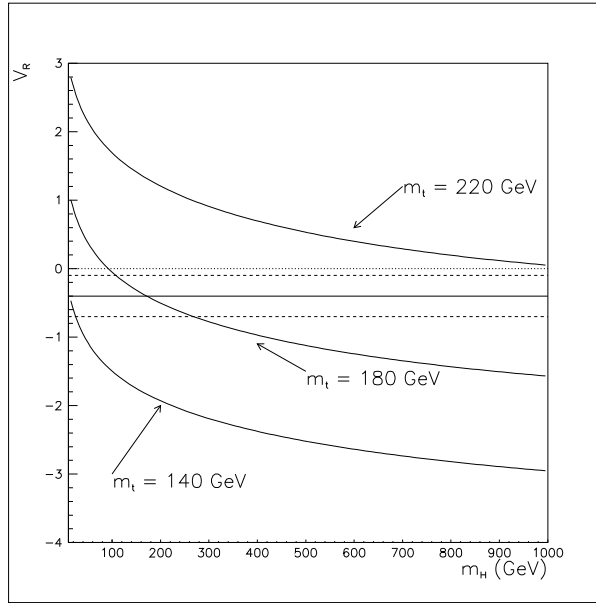
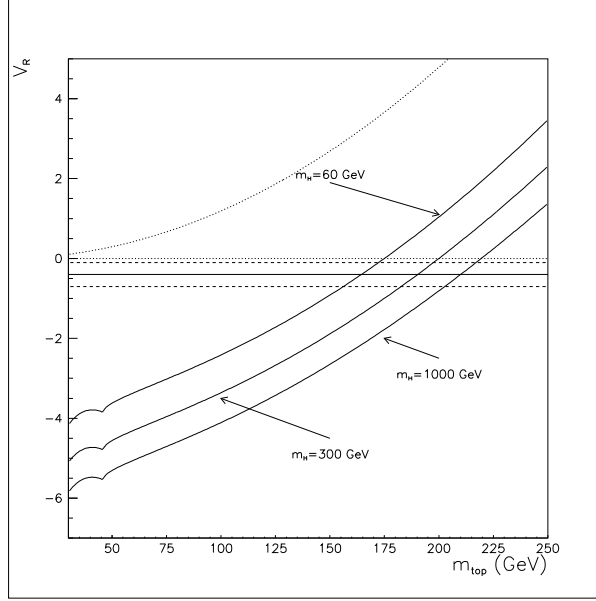


Figure 11:  $V_R$  as a function of  $m_t$ . (The remaining clarifications are similar to those given to figure 9) (a).  $V_R$  as a function of  $m_H$  for three values of  $m_t$ : 150, 175 and 200 GeV. (See caption to figure 9) (b).



independent of both  $t$  and  $h$ ). Therefore the curves for  $m_H = 60$  and  $1000$  GeV in figures 9a, 10a, 11a are (mostly) produced by the parallel transfer of the curve for  $m_H = 300$  GeV.

We see in figures 9a, 10a and 11a that if the  $t$ -quark were light, radiative corrections would be negative, and if it were very heavy, they would be much larger. This looks like a conspiracy of the observable mass of the  $t$ -quark and all other parameters of the electroweak theory, as a result of which the electroweak correction  $V_R$  becomes anomalously small. Note that the corrections do not vanish simultaneously because, if we fix the value of  $m_H$ , then  $V_m(t)$ ,  $V_A(t)$  and  $V_R(t)$  cross the horizontal lines  $V_i = 0$  at different values of  $m_t$ . What happens is an approximate vanishing of the correction, which as if corresponds to some broken symmetry. The nature of this symmetry is not clear at all, and even its existence is very problematic.

One should specially note the dashed parabola in figures 9a, 10a and 11a corresponding to the Veltman term  $t$ . We see that in the interval  $0 < m_t < 250$  GeV it lies much higher than  $V_A$  and  $V_R$  and approaches  $V_m$  only in the right-hand side of figure 9a. Therefore, the so-called non-leading ‘small’ corrections that are typically replaced with ellipses in standard texts, are found to be comparable with the leading term  $t$ .

A glance at figures 9a, 10a, 11a readily explains how the experimental analysis of electroweak corrections allows, despite their smallness, a prediction, within the framework of the minimal standard model, of the  $t$ -quark mass. Even when the experimental accuracy of LEP1 and SLC experiments was not sufficient for detecting electroweak corrections, it was sufficient for establishing the  $t$ -quark mass using the points at which the curves  $V_i(m_t)$  intersect the horizontal lines corresponding to the experimental values of  $V_i$  and the parallel to them thin lines that show the band of one standard deviation. The accuracy in determining  $m_t$  is imposed by the band width and the slope of  $V_i(m_t)$  lines.

The dependence  $V_i(m_H)$  for three fixed values of  $m_t = 150, 175$  and  $200$  GeV (figures 9b, 10b, 11b) can be presented in a similar manner. As follows from the explicit form of the terms  $H_i(m_H)$ , the dependence  $V_i(m_H)$  is considerably less steep (logarithmic). This is the reason why the prediction of the higgs mass extracted from electroweak corrections has such a high uncertainty. We will see later (figures 13, 14, 15) that the accuracy of prediction of  $m_H$  will greatly depend on what the  $t$ -quark’s mass is going to be. If  $m_t = 150 \pm 5$  GeV, then  $m_H < 200$  GeV at the  $3\sigma$  level. If  $m_t = 200 \pm 5$  GeV, then  $m_H > 120$  GeV at the  $3\sigma$  level. If, however,  $m_t = 175 \pm 5$  GeV, we are hugely unlucky: there is practically no constraint on  $m_H$ .

Before starting a discussion of hadronic decays of the  $Z$  boson, let us ‘go back to the roots’ and recall how the equations for  $V_i(m_t, m_H)$  were derived.

### How to calculate $V_i$ ? ‘Five steps’.

An attentive reader should have already come up with the question: what makes the amplitudes of the lepton decays of the  $Z$  boson in the one-loop approximation depend on the self-energy of the  $W$  boson? Indeed, the loops describing the self-energy of the  $W$  boson appear in the decay diagrams of the  $Z$  boson only beginning with the two-loop approximation. The answer to this question is this. We have already emphasized that we find expressions for radiative corrections to  $Z$ -boson decays in terms of  $\bar{\alpha}$ ,  $m_Z$  and  $G_\mu$ . However, the expression for  $G_\mu$  includes the self-energy of the  $W$  boson even in the one-loop approximation. The

point is thus in our expressing some observables (in this particular case,  $m_W/m_Z$ ,  $g_A$ ,  $g_V/g_A$ ) in terms of other, more accurately measured observables ( $\bar{\alpha}$ ,  $m_Z$ ,  $G_\mu$ ).

Let us trace how this is achieved, step by step. There are altogether ‘five steps to happiness’, based on the one-loop approximation.

**Step I.** We begin with the electroweak Lagrangian after it had undergone the spontaneous violation of the  $SU(2) \times U(1)$ -symmetry by the higgs vacuum condensate (vacuum expectation value – VEV)  $\eta$  and the  $W$  and  $Z$  bosons became massive. Let us consider the bare coupling constants (the bare charges  $e_0$  of the photon,  $g_0$  of the  $W$ -boson and  $f_0$  of the  $Z$ -boson) and the bare masses of the vector bosons:

$$m_{Z0} = \frac{1}{2}f_0\eta \quad , \quad (82)$$

$$m_{W0} = \frac{1}{2}g_0\eta \quad , \quad (83)$$

and also bare masses:  $m_{t0}$  of the  $t$ -quark and  $m_{H0}$  of the higgs.

**Step II.** We express  $\bar{\alpha}$ ,  $G_\mu$ ,  $m_Z$  in terms of  $f_0$ ,  $g_0$ ,  $e_0$ ,  $\eta$ ,  $m_{t0}$ ,  $m_{H0}$  and  $1/\varepsilon$  (see Appendix E). Here  $1/\varepsilon$  appears because we use the dimensional regularization, calculating Feynman integrals in the space of  $D$  dimensions (see Appendix A). These integrals diverge at  $D = 4$  and are finite in the vicinity of  $D = 4$ . By definition,

$$2\varepsilon = 4 - D \rightarrow 0 \quad . \quad (84)$$

Note that in the one-loop approximation  $m_{t0} = m_t$ ,  $m_{H0} = m_H$ , since we neglect the electroweak corrections to the masses of the  $t$ -quark and the higgs. For the higgs this approximation is quite legitimate, since the accuracy of extracting its mass from radiative corrections is very poor. As for the  $t$ -quark, this statement is also true at the current accuracy of the experimental measurement of electroweak corrections; however, this would become an unacceptably crude approximation if this accuracy could be improved by an order of magnitude at LEP and SLC. The situation here is analogous to that for  $G_\mu$  and the self-energy of the  $W$ -boson.

Step II is almost physics: we calculate Feynman diagrams (we say ‘almost’ to emphasize that observables are expressed in terms of nonobservable, ‘bare’, and generally infinite quantities).

**Step III.** Let us invert the expressions derived at step II and write  $f_0$ ,  $g_0$ ,  $\eta$  in terms of  $\bar{\alpha}$ ,  $G_\mu$ ,  $m_Z$ ,  $m_t$ ,  $m_H$  and  $1/\varepsilon$ . This step is pure algebra.

**Step IV.** Let us express  $V_m$ ,  $V_A$ ,  $V_R$  (or the electroweak one-loop correction to any other electroweak observable, all of them being treated on an equal basis) in terms of  $f_0$ ,  $g_0$ ,  $\eta$ ,  $m_t$ ,  $m_H$  and  $1/\varepsilon$ . (Like step II, this step is again almost physics.)

**Step V.** Let us express  $V_m$ ,  $V_A$ ,  $V_R$  (or any other electroweak correction) in terms of  $\bar{\alpha}$ ,  $G_\mu$ ,  $m_Z$ ,  $m_t$ ,  $m_H$  using the results of steps III and IV. Formally this is pure algebra, but in fact pure physics, since now we have expressed certain physical observables in terms of other observables. If no errors were made on the way, the terms  $1/\varepsilon$  cancel out. As a result, we arrive at formula (58) which gives  $V_i$  as elementary functions of  $t$ ,  $h$  and  $s$ .

The five steps outlined above are very simple and visually clear. We obtain the main relations without using the ‘heavy artillery’ of quantum field theory with its counterterms in

the Lagrangian and the renormalization procedure. This simplicity and visual clarity became possible owing to the one-loop electroweak approximation (even though this approach to renormalizations is possible in multiloop calculations, it becomes more cumbersome than standard procedures). As for the QCD-corrections to quark electroweak loops and the two-loop higgs contribution hidden in the terms  $\delta V_i$  in equation (58), we take the relevant formulas from the calculations of other authors.

## 5 Hadronic decays of the Z boson. The leading quarks and hadrons.

As discussed above (see formulas (35)–(40) and the subsequent section on ‘Asymmetry’), an analysis of hadronic decays reduces to the calculation of decays to pairs of quarks:  $Z \rightarrow q\bar{q}$ . The key role is played by the concept of leading hadrons that carry away the predominant part of the energy. For example, the  $Z \rightarrow c\bar{c}$  decay mostly produces two hadron jets flying in opposite directions, in one of which the leading hadron is the one containing the  $\bar{c}$ -quark, for example,  $D^- = \bar{c}d$ , and in the other the hadron with the  $c$ -quark, for example,  $D^0 = c\bar{u}$  or  $\Lambda_c^+ = udc$ . Likewise,  $Z \rightarrow b\bar{b}$  decays are identified by the presence of high-energy  $B$  or  $\bar{B}$  mesons. If we select only particles with energy close to  $m_Z/2$ , the identification of the initial quark channels is unambiguous. The total number of such cases will, however, be small. If we take into account as a signal less energetic  $B$ -mesons, we face the problem of their origin. Indeed, a pair  $b\bar{b}$  can be created not only directly by a  $Z$  boson but also by a virtual gluon in, say, a  $Z \rightarrow c\bar{c}$  decay (figure 12a) or  $Z \rightarrow u\bar{u}$ , or  $s\bar{s}$ . This example shows the sort of difficulty encountered by experimentalists trying to identify a specific quark–antiquark channel. Furthermore, owing to secondary pairs, the total hadron width is not strictly equal to the sum of partial quark widths.

We remind the reader that for the partial width of the  $Z \rightarrow q\bar{q}$  decay we had (35)

$$\Gamma_q \equiv \Gamma(Z \rightarrow q\bar{q}) = 12\Gamma_0[g_{Aq}^2 R_{Aq} + g_{Vq}^2 R_{Vq}^2] , \quad (85)$$

where the standard width  $\Gamma_0$  is, according to (31),

$$\Gamma_0 = \frac{G_\mu m_Z^3}{24\sqrt{2}\pi} = 82.944(6)\text{MeV} , \quad (86)$$

and the radiators  $R_{Aq}$  and  $R_{Vq}$  are given in Appendix F. As for the electroweak corrections, they are included in the coefficients  $g_{Aq}$  and  $g_{Vq}$ . The sum of the Born and one-loop terms has the form

$$g_{Aq} = T_{3q}[1 + \frac{3\bar{\alpha}}{32\pi s^2 c^2} V_{Aq}(t, h)] , \quad (87)$$

$$R_q \equiv g_{Vq}/g_{Aq} = 1 - 4|Q_q|s^2 + \frac{3|Q_q|}{4\pi(c^2 - s^2)} \bar{\alpha} V_{Rq}(t, h) . \quad (88)$$

### Decays to pairs of light quarks.

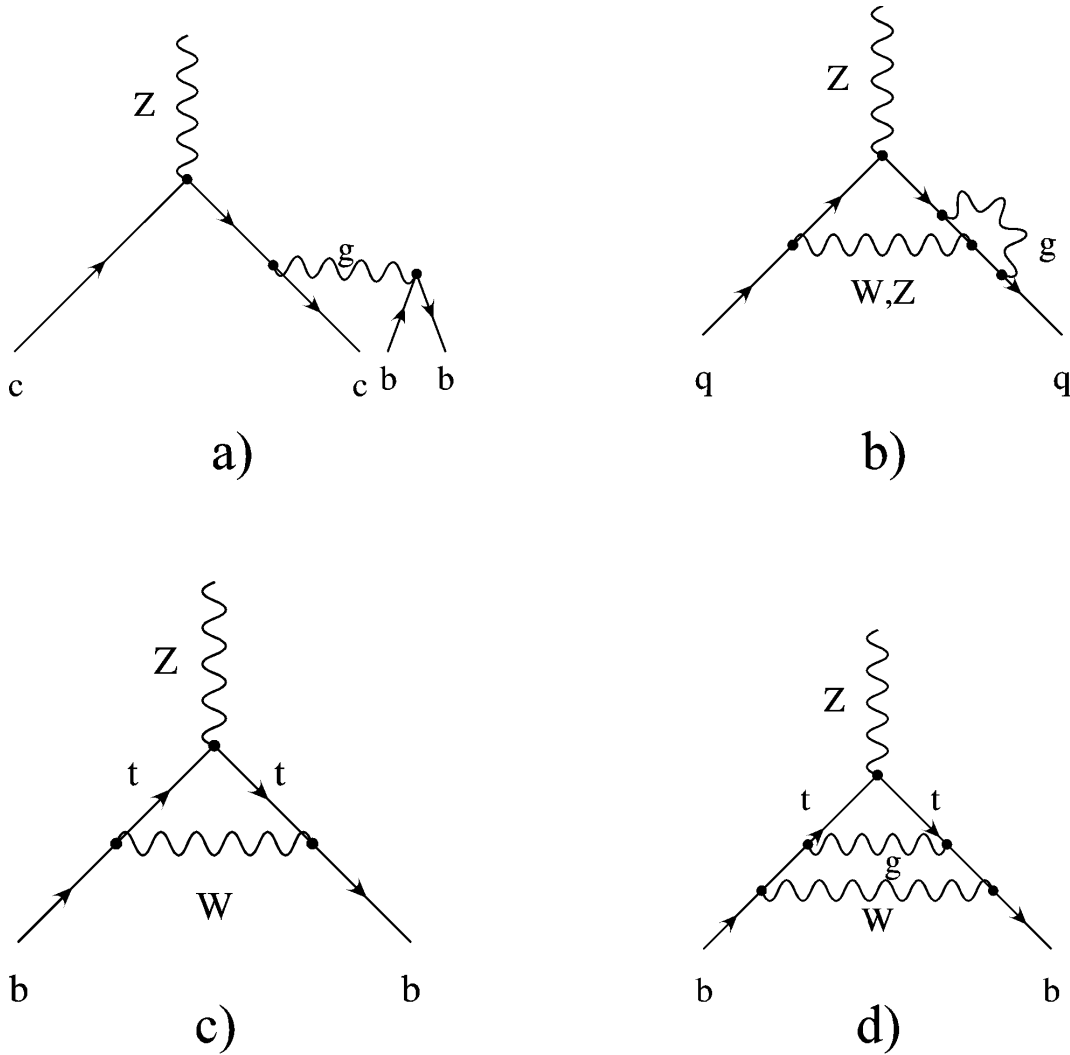


Figure 12: The  $Z \rightarrow c\bar{c}$  decay producing a secondary pair  $b\bar{b}$  created by a virtual gluon  $g$  (a). The  $Z \rightarrow q\bar{q}$  decay with a virtual gluon  $g$  which connects a side of the quark triangle with the external quark line. Diagrams of this type have not been calculated yet (b). The verted electroweak diagram involving  $t$ -quarks and contributing to the  $Z \rightarrow b\bar{b}$  decay (c). One of the diagrams describing gluon corrections to the diagram of figure 12(c).

Here, as in the case of hadronless observables, the quantities  $V$  that characterize corrections are normalized in the standard way:  $V \rightarrow t$  as  $t \gg 1$ . Naturally, those terms in  $V$  that are due to the self-energies of vector bosons are identical for both leptons and quarks. The deviation of the differences  $V_{Aq} - V_A$  and  $V_{Rq} - V_R$  from zero are caused by the differences in radiative corrections to vertices  $Z \rightarrow q\bar{q}$  and  $Z \rightarrow l\bar{l}$ . For four light quarks we have

$$V_{Au}(t, h) = V_{Ac}(t, h) = V_A(t, h) + \frac{128\pi s^3 c^3}{3\bar{\alpha}}(F_{Al} + F_{Au}) , \quad (89)$$

$$V_{Ad}(t, h) = V_{As}(t, h) = V_A(t, h) + \frac{128\pi s^3 c^3}{3\bar{\alpha}}(F_{Al} - F_{Ad}) , \quad (90)$$

$$V_{Ru}(t, h) = V_{Rc}(t, h) = V_R(t, h) + \frac{16\pi s c(c^2 - s^2)}{3\bar{\alpha}} \times \\ \times [F_{Vl} - (1 - 4s^2)F_{Al} + \frac{3}{2}(- (1 - \frac{8}{3}s^2)F_{Au} + F_{Vu})] , \quad (91)$$

$$V_{Rd}(t, h) = V_{Rs}(t, h) = V_R(t, h) + \frac{16\pi s c(c^2 - s^2)}{3\bar{\alpha}} \times \\ \times [F_{Vl} - (1 - 4s^2)F_{Al} + 3((1 - \frac{4}{3}s^2)F_{Ad} - F_{Vd})] , \quad (92)$$

where (see Appendix M):

$$F_{Al} = \frac{\bar{\alpha}}{4\pi}(3.0099 + 16.4\delta s^2) , \quad (93)$$

$$F_{Vl} = \frac{\bar{\alpha}}{4\pi}(3.1878 + 14.9\delta s^2) , \quad (94)$$

$$F_{Au} = -\frac{\bar{\alpha}}{4\pi}(2.6802 + 14.7\delta s^2) , \quad (95)$$

$$F_{Vu} = -\frac{\bar{\alpha}}{4\pi}(2.7329 + 14.2\delta s^2) , \quad (96)$$

$$F_{Ad} = \frac{\bar{\alpha}}{4\pi}(2.2221 + 13.5\delta s^2) , \quad (97)$$

$$F_{Vd} = \frac{\bar{\alpha}}{4\pi}(2.2287 + 13.5\delta s^2) . \quad (98)$$

The values of  $F$  are given here for  $s^2 = 0.23110 - \delta s^2$ . The accuracy to five decimal places is purely arithmetic. The physical uncertainties introduced by neglecting higher-order loops manifest themselves already in the third decimal place. It is necessary to point out that corrections of the type of that shown in figure 12b have not yet been calculated.

### Decays to $b\bar{b}$ pair.

In the  $Z \rightarrow b\bar{b}$  decay it is necessary to take into account additional  $t$ -dependent vertex corrections:

$$V_{Ab}(t, h) = V_{Ad}(t, h) - \frac{8s^2 c^2}{3(3 - 2s^2)}(\phi(t) + \delta\phi(t)) , \quad (99)$$

$$V_{Rb}(t, h) = V_{Rd}(t, h) - \frac{4s^2(c^2 - s^2)}{3(3 - 2s^2)}(\phi(t) + \delta\phi(t)). \quad (100)$$

Here the term  $\phi(t)$  calculated in [57] corresponds to figure 12c and the term  $\delta\phi(t)$  calculated in [58], [53] corresponds to the leading gluon and higgs corrections to the term  $\phi(t)$  (see figure 12d). Expressions for  $\phi(t)$  and  $\delta\phi(t)$  are given in Appendix N. For  $m_t = 175$  GeV,  $\hat{\alpha}_s(m_Z) = 0.125$ ,  $m_H = 300$  GeV,

$$\phi(t) = 29.9 \quad , \quad (101)$$

$$\delta\phi(t) = -3.0 \quad , \quad (102)$$

and correction terms in equations (99) and (100) are very large: they equal  $-5.0$  and  $-1.8$ , respectively.

## 6 Comparison of theoretical results and experimental LEP1 and SLC data.

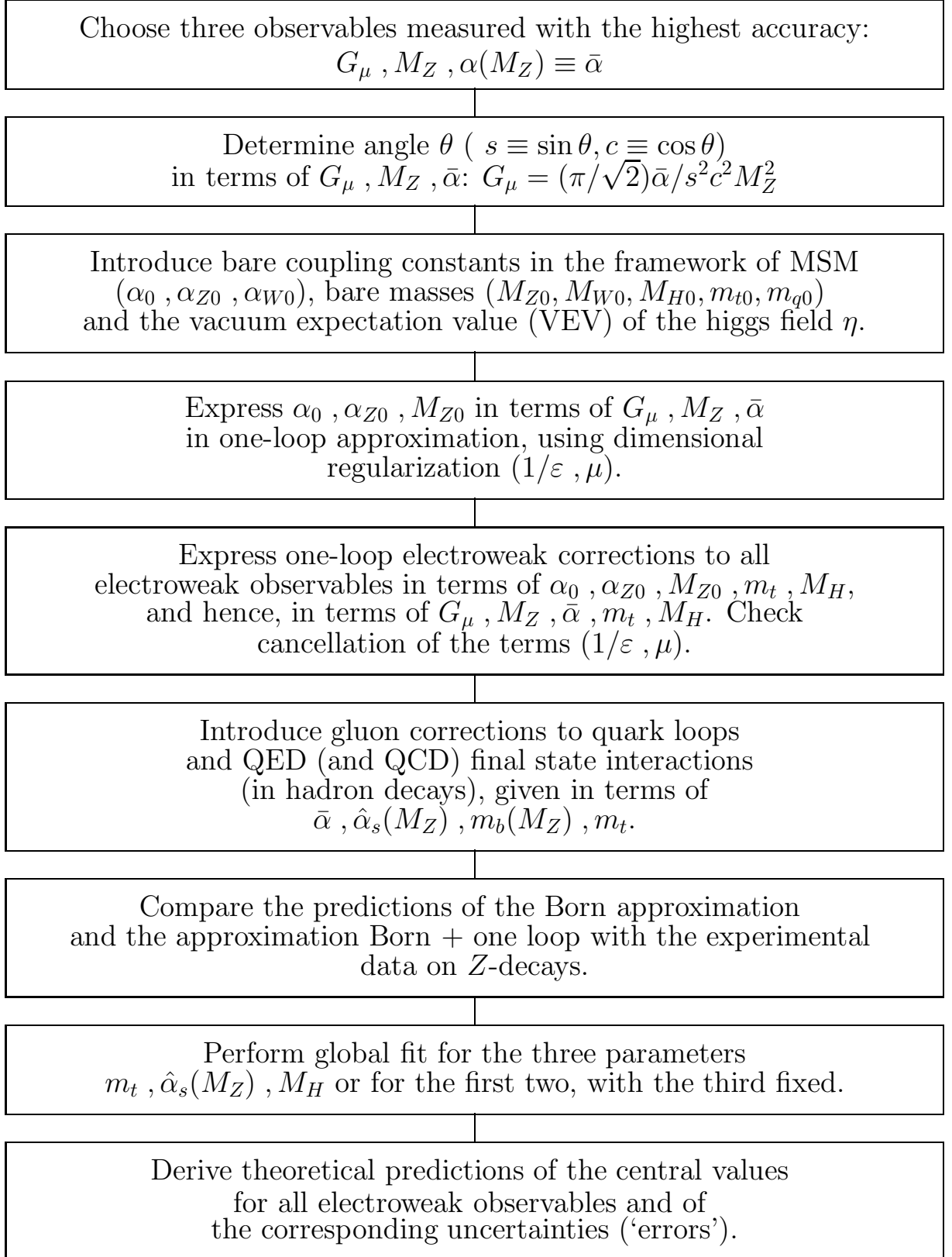
### LEPTOP code.

A number of computer programs (codes) were written for comparing high-precision data of LEP1 and SLC. The best known of these programs in Europe is ZFITTER [59], which takes into account not only electroweak radiative corrections but also all purely electromagnetic ones, including, among others, the emission of photons by colliding electrons and positrons. Some of the first publications in which the  $t$  quark mass was predicted on the basis of precision measurements [60], were based on the code ZFITTER. Other European codes, BHM [61], WOH, TOPAZO [62], somewhat differ from ZFITTER. The best known in the USA are the results generated by the code used by Langacker [35].

The original idea of the authors of this review in 1991–1993 was to derive simple analytical formulas for electroweak radiative corrections, which would make it possible to predict the  $t$ -quark mass using no computer codes, just by analyzing experimental data on a sheet of paper. Alas, the diversity of hadron decays of  $Z$  bosons, depending on the constants of strong gluon interaction  $\hat{\alpha}_s$ , was such that it was necessary to convert analytical formulas into a computer program which we jokingly dubbed LEPTOP [63]. The LEPTOP calculates the electroweak observables in the framework of the Minimal Standard Model and fits experimental data so as to determine the quantities  $m_t$ ,  $M_H$  and  $\hat{\alpha}_s(M_Z)$ . The logical structure of LEPTOP is clear from the preceding sections of this review and is shown in the Flowchart. The code of LEPTOP can be downloaded from the Internet home page: [http://cppm.in2p3.fr/leptop/intro\\_leptop.html](http://cppm.in2p3.fr/leptop/intro_leptop.html)

A comparison of the codes ZFITTER, BHM, WOH, TOPAZO and LEPTOP carried out in 1994–95 [18] has demonstrated that their predictions for all electroweak observables coincide with accuracy that is much better than the accuracy of the experiment. The results of processing the experimental data using LEPTOP are shown below.

### Flowchart.



### General fit.

Table 5 shows experimental values of the electroweak observables, obtained by averaging the results of four LEP detectors (part a), and also SLC data (part b) and the data on  $W$  boson mass (part c). (The data on the  $W$  boson mass from the  $p\bar{p}$ -colliders are also shown, for the reader's convenience, in the form of  $s_W^2$ , while the data on  $s_W^2$  from  $\nu N$ -experiments are also shown in the form of  $m_W$ . These two numbers are given in italics, emphasizing that they are not independent experimental data.) We take experimental data from the paper [44].

Table 5 sums up the experimental data used for determining (fitting) the parameters of the standard model  $m_t$  and  $\hat{\alpha}_s(m_Z)$  (see Table 7). The central values in the third column of Table 5 were calculated for  $m_H = 300$  GeV. Shown in brackets is the uncertainty of the last significant decimal places due to the uncertainty of the fitted values  $m_t$  and  $\hat{\alpha}_s$ . Above and below we give the shifts in the last significant decimal places corresponding to  $m_H = 1000$  GeV and  $m_H = 60$  GeV, respectively. The last column shows the value of the ‘pull’. By definition, the pull is the difference between the experimental and the theoretical values divided by experimental uncertainty. The pull values show that the maximum discrepancy between the experimental data and the Minimal Standard Model is found for  $R_b$  ( $3.8\sigma$ ). Deviations at the level of  $2.5\sigma$  are also found for  $R_c$  and  $\sin^2 \theta_{eff}^{lept} \equiv s_l^2$  in  $A_{LR}$  (SLC). For most observables the discrepancy is less than  $1\sigma$ . At the same time, Table 6 shows that the value  $s_l^2 = 0.23186(34)$  extracted from all asymmetries at LEP agrees quite well with the fitted MSM value  $s_l^2 = 0.2321(4)$  from the LEP data, and for all sets of data.



**Table 5**

Observable	Experimental data	Fit standard model	Pull
a) <u>LEP</u>			
shape of $Z$ -peak and lepton asymmetries:			
$m_Z$ [GeV]	91.1884(22)		
$\Gamma_Z$ [GeV]	2.4963(32)	$2.4976(26)^{+6}_{-16}$	-0.4
$\sigma_h[nb]$	41.488(78)	$41.450(20)^{+3}_{-7}$	0.5
$R_l$	20.788(32)	$20.770(24)^{-5}_{+11}$	0.6
$A_{FB}^l$	0.0172(12)	$0.0158(6)^{-2}_{+3}$	1.2
$\tau$ -polarization:			
$A_\tau$	0.1418(75)	$0.1450(26)^{-7}_{+13}$	-0.4
$A_e$	0.1390(89)	$0.1450(26)^{-7}_{+13}$	-0.7
Results for $b$ and $c$ quarks:			
$R_b$	0.2219(17)	$0.2155(3)^{-7}_{+7}$	3.8
$R_c$	0.1543(74)	$0.1724(1)^{+2}_{-2}$	-2.5
$A_{FB}^b$	0.0999(31)	$0.1017(19)^{-6}_{+10}$	-0.6
$A_{FB}^c$	0.0725(58)	$0.0726(14)^{-4}_{+7}$	0.0
Charge asymmetry for pairs of light quarks $q\bar{q}$ :			
$s_l^2(< Q_{FB} >)$	0.2325(13)	$0.2318(3)^{+1}_{-2}$	0.6
b) <u>SLC</u>			
$A_{LR}$	0.1551(40)	$0.1450(26)^{-7}_{+13}$	2.5
$s_l^2(A_{LR})$	$0.2305(5)$	$0.2318(3)^{+1}_{-2}$	-2.5
$R_b$	0.2171(54)	$0.2155(3)^{-7}_{+7}$	0.3
$A_b$	0.8410(530)	$0.9345(2)^{-3}_{+4}$	-1.8
$A_c$	0.6060(900)	$0.6670(12)^{-3}_{+6}$	-0.7
c) <u><math>p\bar{p}</math> and <math>\nu N</math></u>			
$m_W$ [GeV] ( $p\bar{p}$ )	80.26(16)	$80.35(5)^{+1}_{-3}$	-0.5
	$0.2253(31)$		
$1 - m_W^2/m_Z^2$ ( $\nu N$ )	0.2257(47)	$0.2237(9)^{-2}_{+5}$	0.4
	$80.24(24)$		

**Table 6**

Observable	$s_l^2$	Average over groups of observations	Cumulative average	$\chi^2/d.o.f.$
$A_{FB}^l$	0.23096(68)	0.23160(49)	0.23160(49)	1.9/2
$A_\tau$	0.23218(95)			
$A_e$	0.2325(11)			
$A_{FB}^b$	0.23209(55)	0.23205(51)	0.23182(35)	2.4/4
$A_{FB}^c$	0.2318(13)			
$\langle Q_{FB} \rangle$	0.2325(13)	0.2325(13)	0.23186(34)	2.6/5
$A_{LR}$ (SLD)	0.23049(50)	0.23049(50)	0.23143(28)	7.8/6

Table 6 gives experimental values of  $s_l^2$ . The third column was obtained by averaging of the second column, and the fourth by cumulative averaging of the third; it also lists the values of  $\chi^2$  over the number of degrees of freedom.

**Table 7**

Physical quantities	LEP	LEP + SLC	LEP + $M_W$	LEP + SLC + $M_W$
$m_t$ (GeV)	171(9) $^{+18}_{-21}$	182(7) $^{+18}_{-22}$	170(8) $^{+17}_{-21}$	180(7) $^{+18}_{-21}$
$\hat{\alpha}_s$	0.125(4) $^{+2}_{-2}$	0.123(4) $^{+2}_{-2}$	0.125(4) $^{+2}_{-2}$	0.124(4) $^{+2}_{-2}$
$\chi^2/d.o.f.$	18/9	29/13	18/11	30/15
$s_l^2$	0.2321(4) $^{+1}_{-2}$	0.2317(3) $^{+1}_{-2}$	0.2321(4) $^{+1}_{-2}$	0.2318(3) $^{+1}_{-2}$
$g_{Vl}/g_{Al}$	0.07116(16) $^{+4}_{-8}$	0.0732(12) $^{+4}_{-8}$	0.0716(16) $^{+4}_{-8}$	0.0728(12) $^{+4}_{-8}$
$s_W^2$	0.2247(9) $^{+2}_{-4}$	0.2234(9) $^{+2}_{-5}$	0.2237(9) $^{+2}_{-4}$	0.2237(9) $^{+2}_{-5}$
$m_W/m_Z$	0.8805(5) $^{+1}_{-2}$	0.8813(5) $^{+1}_{-3}$	0.8804(5) $^{+1}_{-2}$	0.8811(5) $^{+2}_{-3}$
$m_W$ (GeV)	80.29(5) $^{+1}_{-2}$	80.36(5) $^{+1}_{-3}$	80.28(5) $^{+1}_{-2}$	80.35(5) $^{+1}_{-3}$

Table 7 gives the fitted values  $m_t$  and  $\hat{\alpha}_s \equiv \hat{\alpha}_s(m_Z^2)$ , and also the values of  $\chi^2$  times the number of degrees of freedom for various sets of data, where  $M_W$  stands for both the data of  $M_W$  measurements in  $p\bar{p}$  collisions and the values of  $s_W^2$ , extracted from experiments with  $\nu N$ . The lower part of the table gives the values of  $s_l^2 \equiv \sin^2 \theta_{eff}^{lept} \equiv \frac{1}{4}(1 - g_{Vl}/g_{Al})$  and  $s_W^2 \equiv 1 - m_W^2/m_Z^2$ , calculated in one-loop electroweak approximation in the framework of the Minimal Standard Model using fitted values of  $m_t$  and  $\hat{\alpha}_s$ . Errors given in parentheses are due to uncertainties in  $m_t$ ,  $\hat{\alpha}_s$  and  $\bar{\alpha}$ , and were calculated by summation of squares, ignoring correlations. Note that the errors in the values of  $s_W^2$  calculated using the fits are substantially lower than in the experimental values of this quantity (see Table 5); at the same time, the errors for  $s_l^2$  are practically identical for the experimental (Table 6) and the theoretical (Table 7) values. Note that the first and second rows of the lower part of Table 7 carry identical information; the same is true for the third, fourth and fifth rows.

## 7 Conclusions.

### Achievements.

What are the main results obtained with four detectors of the LEP1 collider and one detector of the SLC?

Judging by the criteria of accelerator and experimental techniques, the highest imaginable level has been achieved in the team creativity. The impossible became the reality owing to a never before dreamt-of sophistication of equipment of gigantic high-precision detectors. Twenty million decays of  $Z$  bosons were measured with better accuracy than that admired in gemstone cutting.

From the physics standpoint, the main result is the experimental proof that there exist only three generations of quarks and leptons with light neutrinos. The number of light neutrinos, found from the sum width of the invisible  $Z$  boson decays is

$$N_\nu = 2.990(16) \quad (103)$$

The lower limit on the masses of the heavy neutrinos in the additional generations, provided they exist, is close to  $m_Z/2$  and equals 44 GeV.

No new particles were found in  $Z$  boson decays. In particular, the higgs was not found. From LEP1 data, the lower limit on the higgs mass is

$$m_H > 60 \text{ GeV} \quad (104)$$

The high-precision measurement of the  $Z$  boson mass, its total and partial decay widths and also of the  $P$ - and  $C$ -violating asymmetries made it possible to experimentally determine the electroweak radiative corrections. A comparison of these experimental values with the results of theoretical calculations based on the Minimal Standard Model led to prediction of the  $t$ -quark mass  $m_t$  and the constant of strong interaction for gluons  $\hat{\alpha}_s$ :

$$m_t = 180(7)_{-21}^{+18} \text{ GeV} \quad (105)$$

$$\hat{\alpha}_s = 0.124(4)_{-2}^{+2} \quad (106)$$

Shown in parentheses here is the uncertainty (one standard deviation) due to the uncertainty of the experimental data. The central value corresponds to the assumption that  $m_H = 300$  GeV, and the upper and lower ‘shifts’ correspond to  $m_H = 1000$  GeV and 60 GeV, respectively. Radiative corrections depend only weakly on  $m_H$ , so one cannot extract the value of  $m_H$  from them. The  $t$ -quark mass predicted on the basis of the radiative corrections within the currently known experimental uncertainties is in excellent agreement with the results of direct measurements of  $m_t$  in CDF and D0 detectors at the Tevatron <sup>1</sup>:

$$m_t = 176(13) \text{ GeV} \quad [13] \text{ CDF} \quad (107)$$

---

<sup>1</sup> At the spring 1996 conferences more accurate data have been presented:

$$m_t = 175.6 \pm 5.7 \pm 7.4 \text{ GeV} \quad \text{CDF}[64]$$

$$m_t = 170 \pm 15 \pm 10 \text{ GeV} \quad \text{D0}[65]$$

$$m_t = 199(30) \text{ GeV} \quad [14] \text{ D0} . \quad (108)$$

After the  $t$ -quark mass uncertainty is further reduced, radiative corrections can be used for determining the region in which the higgs mass can lie. Figures 13–15 show that the results will greatly depend on luck. If  $m_t = 150(5) \text{ GeV}$ , figure 13 demonstrates that  $m_H < 150 \text{ GeV}$  at the  $3\sigma$  level. If  $m_t = 200(5) \text{ GeV}$ , then  $m_H > 120 \text{ GeV}$  at the  $3\sigma$  level (figure 15). If, however,  $m_t = 175(5)$ , then the higgs can have any mass within  $3\sigma$ .

### Problems.

A cursory glance at Table 5 is sufficient for identifying the main problem of the  $Z$ -boson physics: the discrepancy between the measured width of decay into a pair  $b\bar{b}$  and its theoretical prediction.

The supersymmetrization of the standard model may help solving this problem [66] (see figure 16a, this diagram with superpartners increases  $R_b$ ). Theories with extra neutral vector boson  $Z'$  also may be relevant. Let us turn now to  $\hat{\alpha}_s(m_Z)$ . A number of papers [67], [68] pointed out that the value  $0.124(4)_{-2}^{+2}$ , shown in Table 7, is in contradiction with the measurements of  $\hat{\alpha}_s(q^2)$  for  $q^2 \lesssim (10\text{GeV})^2$  in deep inelastic scattering [69], in hadron decays of the  $\Upsilon$ -meson [70] and especially in the spectrum of upsilononium levels [71]. If the low-energy values  $\hat{\alpha}_s(q^2 \lesssim (10\text{GeV})^2)$  are extrapolated in the framework of the standard QCD to  $q^2 = m_Z^2$ , we find  $\hat{\alpha}_s(m_Z^2) = 0.110 - 0.118$ . As for the uncertainty in this range, the authors of [68]–[71] do not come to a common opinion. The most cautious evaluate it as  $\pm 0.005$  [69]–[70]. The bravest one insists on  $\pm 0.001$  [68], [71]. In the last case, there is an obvious contradiction with the value derived by analyzing the  $Z$  boson decay data. This contradiction served as a basis for hypotheses [67] that MSM predictions for hadron widths are modified by the contribution of some new unknown particles to electroweak radiative corrections, for example, squarks and gluino, that is, the supersymmetric partners of quarks and gluons. For loops with these particles to result in sufficiently strong deviations from MSM, it is necessary that squarks and gluino were sufficiently light, with masses of the order of 100 GeV. A deviation of the observable value of  $R_b$  by more than  $3\sigma$  from the values predicted in MSM on the basis of the global fit (see Table 5) is also considered as an independent argument in favor of the view that the new physics ‘lurks round the corner’. Two other deviations from MSM in Table 5 are less serious:  $R_c$  and  $A_{LR}$  are off by 2.5 standard deviations. Note that in the latter case we witness a discrepancy not only with MSM but also with the measurements of the lepton asymmetries at LEP1, since, according to equation (47),  $A_{LR} = A_l$ .

Various parametrizations of the manifestations of the new physics can be found in the literature. The better known ones are the parameters  $S, T, U$  [72] and  $\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_b$  [73].

### Prospects.

The LEP collider completed its work in the LEP1 mode in autumn 1995 and began working in the LEP2 mode. It is expected that the total energy of the electron and positron collision will be raised to 192 GeV. What are the main goals of LEP2 [74]?

A careful measurement of the cross section of the reaction  $e^+e^- \rightarrow W^+W^-$  needed to measure the  $W$ -boson mass with accuracy of the order of 50 MeV and to test whether the

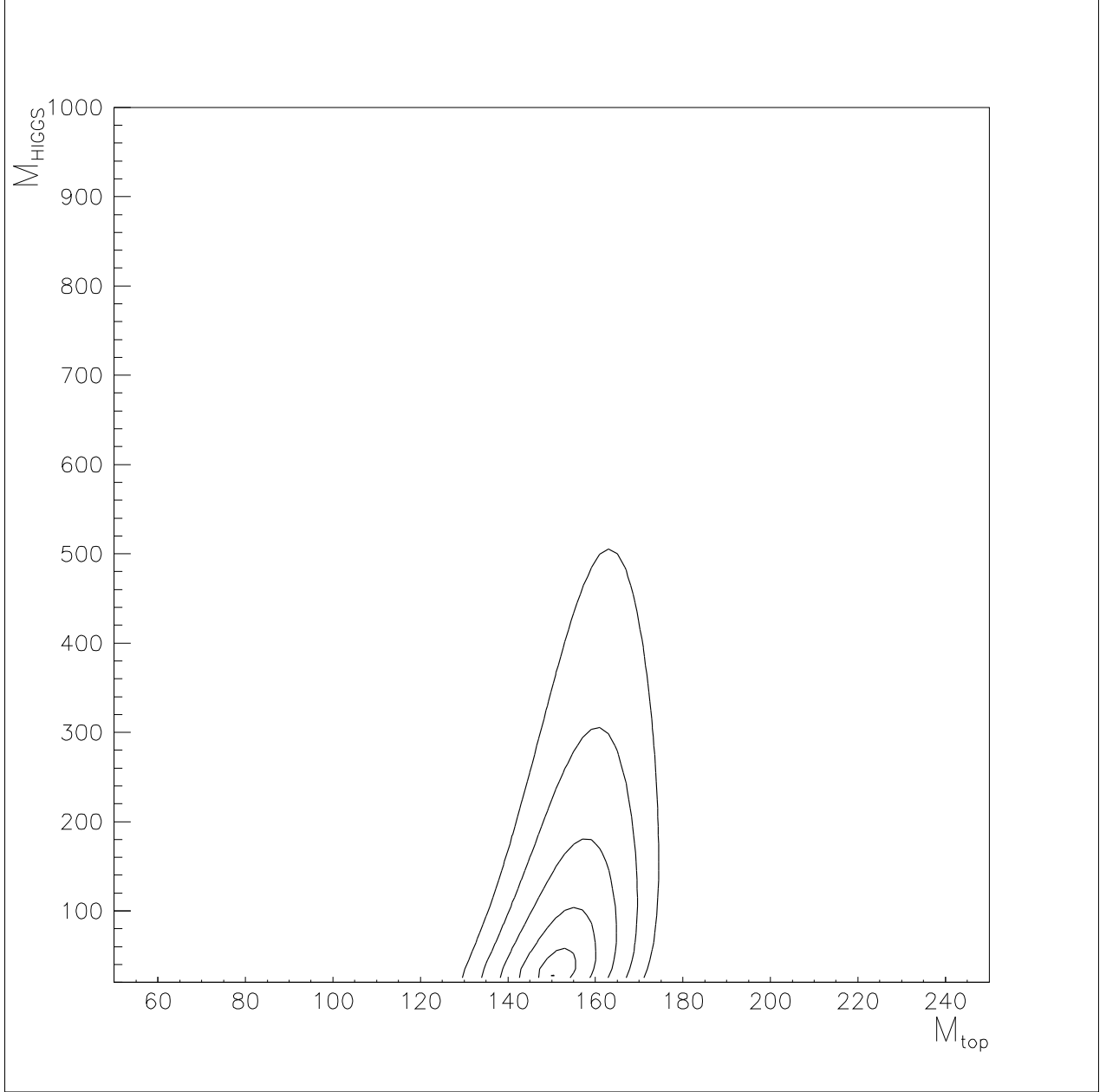


Figure 13: Isolines of  $\chi^2$  in the  $m_t, m_h$  plane, obtained by fitting the electroweak corrections under the assumption that direct measurements of the  $t$  quark mass will give  $m_t = 150 \pm 5$  GeV .

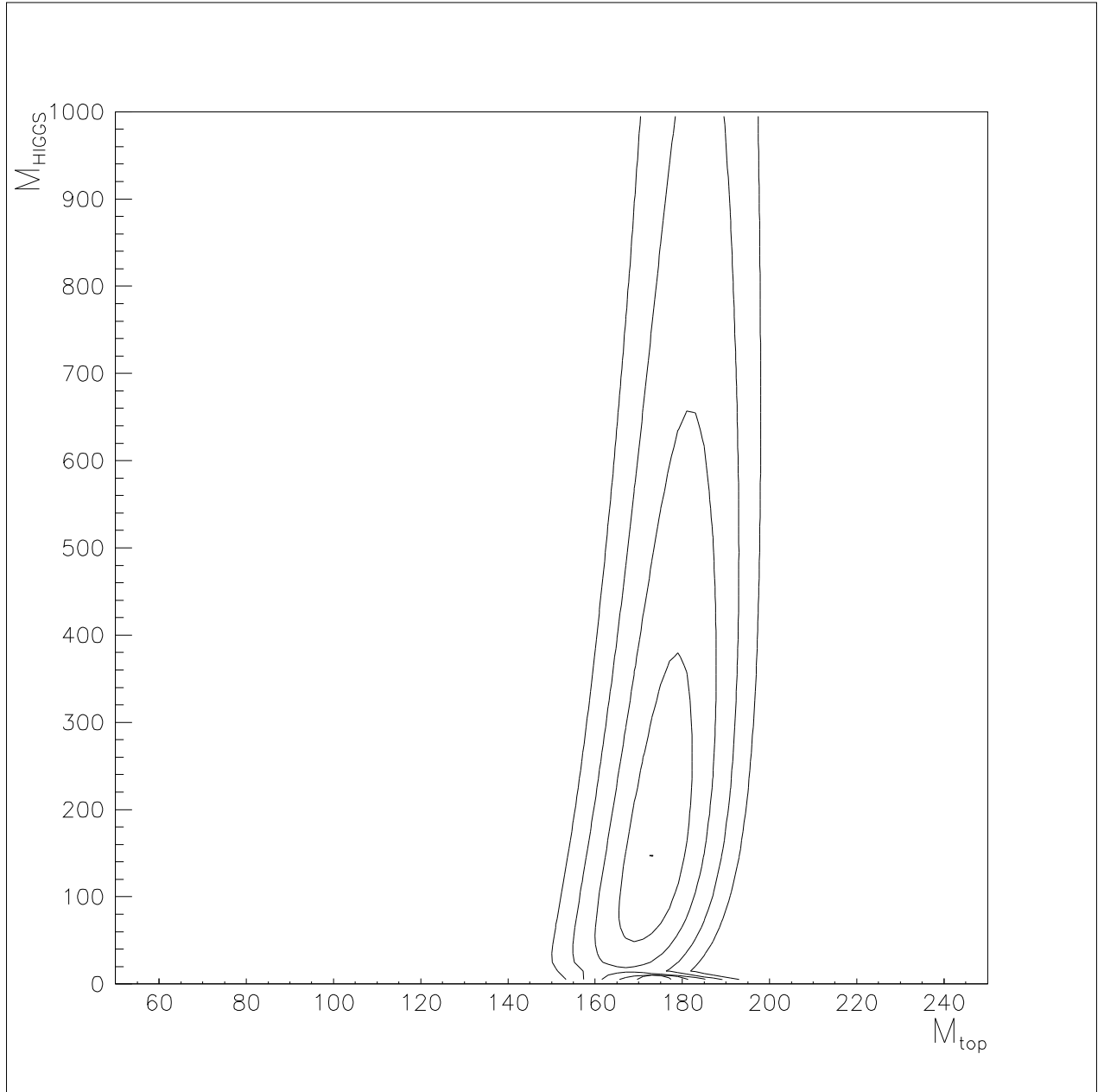


Figure 14: Same as in figure 13, for  $m_t = 175 \pm 5$  GeV.

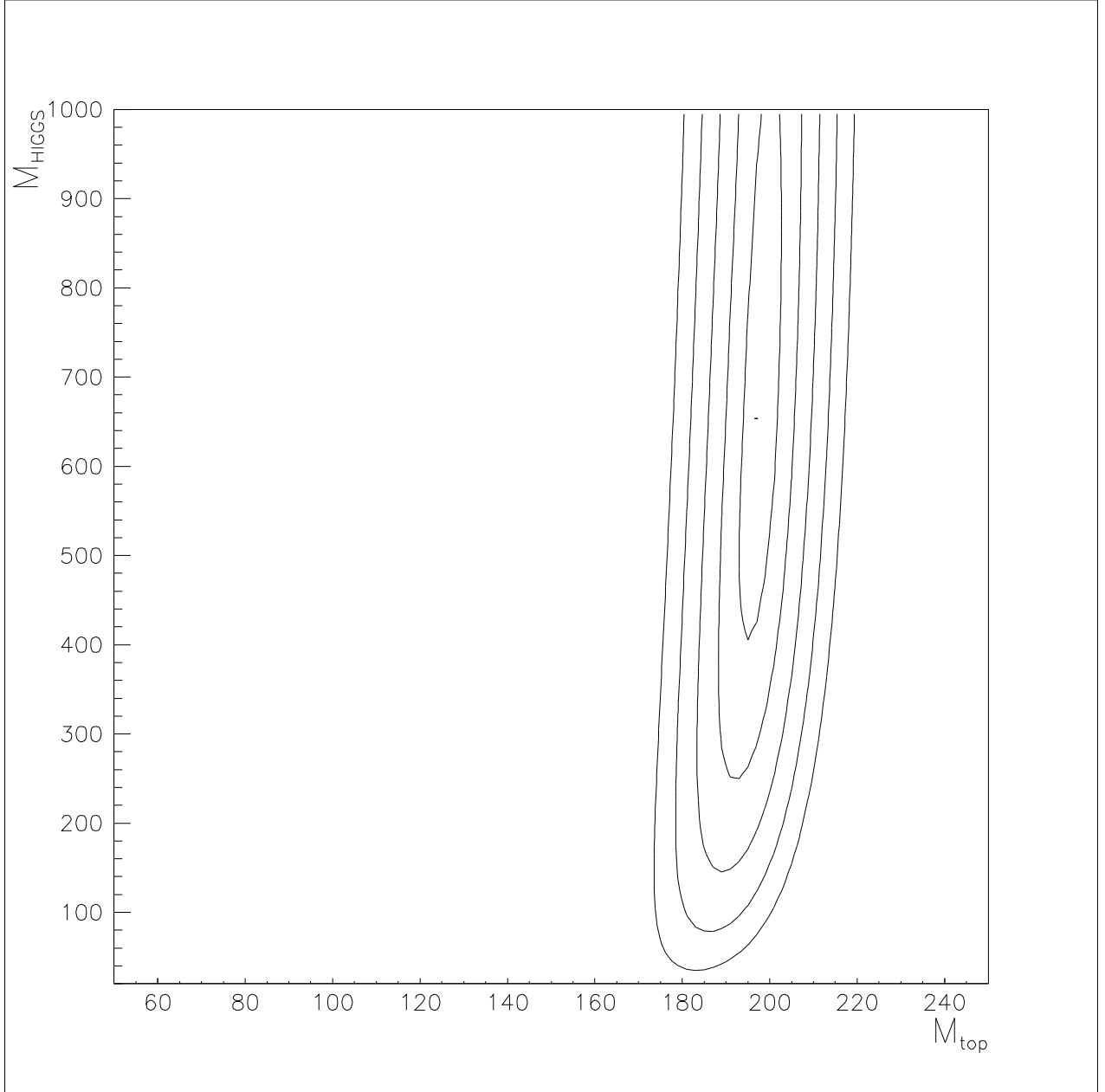


Figure 15: Same as in figure 13, for  $m_t = 200 \pm 5$  GeV.

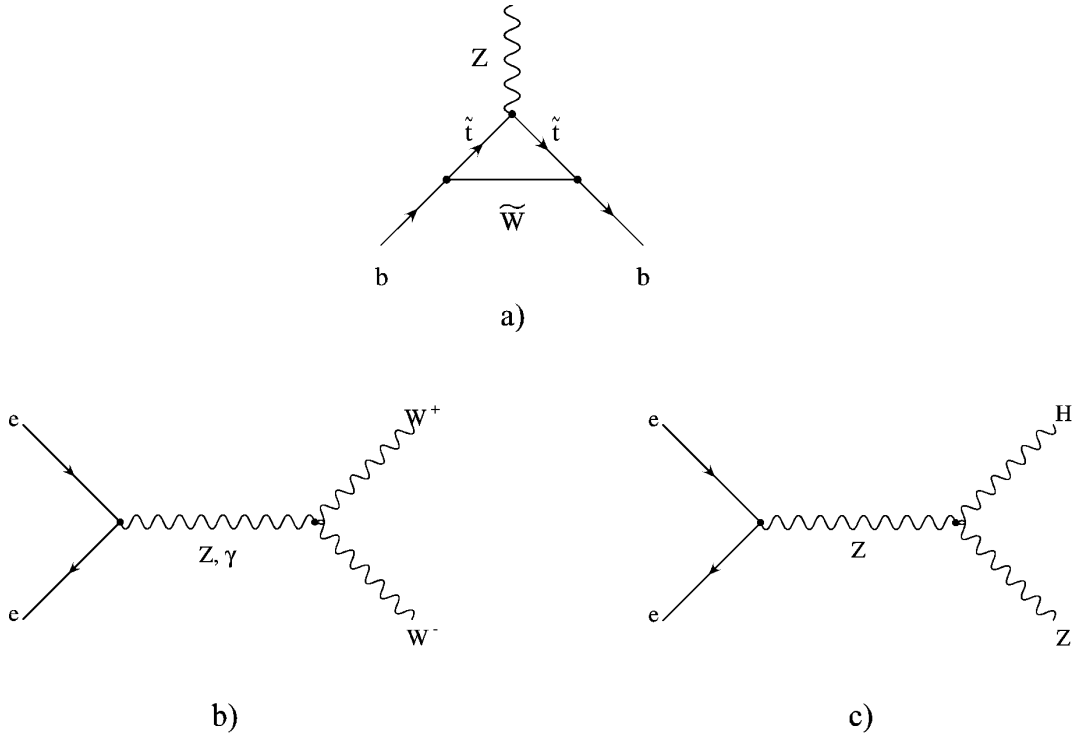


Figure 16: A vertex with virtual  $\tilde{t}$  squarks and winos,  $\tilde{W}$  (a). The reaction  $e^+e^- \rightarrow W^+W^-$  with a virtual photon or  $Z$  boson (b). The reaction  $e^+e^- \rightarrow HZ$  (c).



interaction of  $W$  bosons with photons and with the  $Z$  boson agrees with the standard model (see figure 16b).

A search for a higgs with a mass up to 92 GeV in the reaction  $e^+e^- \rightarrow HZ$  (figure 16c).

A search for light superparticles (sleptons, squarks).

A search for the unanticipated.

Further prospects for testing the Standard Model and for searching for ‘new physics’ beyond its limits are rooted in the Large Hadron Collider (LHC) (a decision to build it has already been made at CERN) and in the so-called ‘Next Linear Collider’ for electrons and positrons, which is so far at the stage of discussion of competing projects.

We are grateful to A. V. Novikov for help in preparing this review; A. R. is grateful to CNRS-IN 2 P3/CPPM for support; M.V., V.N. and L.O. are grateful to RFBR for the grant 93-02-14431; M.V. and V.N. are grateful to INTAS for the grants 93-3316 and 94-2352, which helped to carry out the work presented here.

## Appendix A.

### Feynman rules in electroweak theory.

A consistent derivation of Feynman rules for theories with spontaneous violation of gauge symmetry can be found in textbooks (see, for example, Itzykson, Zuber; Ramond; Slavnov, Faddeev [4]). In this appendix we only give a summary of results, accompanied by brief comments.

#### Gauges and propagators.

Quantization of gauge fields (in MSM this is  $W_\mu^\pm$ ,  $Z_\mu$  and  $A_\mu$ ) requires fixing gauge. The most popular is the so-called  $R_\xi$  gauge which corresponds to adding gauge fixing new terms  $\delta\mathcal{L}_{GF}$  to the classical Lagrangian:

$$\begin{aligned} \delta\mathcal{L}_{GF} = & -\frac{1}{2\xi_A}(\partial_\mu A_\mu)^2 - \frac{1}{2\xi_Z}(\partial_\mu Z_\mu - m_Z\xi_Z G^0)^2 - \\ & -\frac{1}{\xi_W}(\partial_\mu W_\mu^+ - im_W\xi_W G^+)(\partial_\mu W_\mu^- + im_W\xi_W G^-) , \end{aligned} \quad (\text{A.1})$$

where  $G^\pm, G^0$  and  $H$  are the components of the higgs doublet  $\Phi$  in the parametrization

$$\Phi = \begin{pmatrix} G^+(x) \\ \frac{1}{\sqrt{2}}(\eta + H(x) + iG^0(x)) \end{pmatrix} \quad (\text{A.2})$$

In what follows we use the particular case of  $R_\xi$  gauge, namely

$$\xi_A = \xi_W = \xi_Z = \xi$$

With gauge fixed, it is possible to determine the propagators  $D_{\mu\nu}^W(p)$ ,  $D_{\mu\nu}^Z(p)$  and  $D_{\mu\nu}^A(p)$  of the fields  $W_\mu^\pm$ ,  $Z_\mu$  and  $A_\mu$ :

$$\begin{aligned} D_{\mu\nu}^W(p) = & -\frac{i}{p^2 - m_W^2 + i\varepsilon} \left\{ g_{\mu\nu} - (1 - \xi) \frac{p_\mu p_\nu}{p^2 - m_W^2 \xi + i\varepsilon} \right\} \equiv \\ \equiv & -\frac{i}{p^2 - m_W^2 + i\varepsilon} \left( g_{\mu\nu} - \frac{p_\mu p_\nu}{m_W^2} \right) - i \frac{p_\mu p_\nu}{m_W^2} \frac{1}{p^2 - m_W^2 \xi + i\varepsilon} , \end{aligned} \quad (\text{A.3})$$

$$\begin{aligned} D_{\mu\nu}^Z(p) = & -\frac{i}{p^2 - m_Z^2 + i\varepsilon} \left\{ g_{\mu\nu} - (1 - \xi) \frac{p_\mu p_\nu}{p^2 - m_Z^2 \xi + i\varepsilon} \right\} = \\ = & -\frac{i}{p^2 - m_Z^2 + i\varepsilon} \left( g_{\mu\nu} - \frac{p_\mu p_\nu}{m_Z^2} \right) - i \frac{p_\mu p_\nu}{m_Z^2} \frac{1}{p^2 - m_Z^2 \xi + i\varepsilon} , \end{aligned} \quad (\text{A.4})$$

$$D_{\mu\nu}^A(p) = -\frac{i}{p^2 + i\varepsilon} \{g_{\mu\nu} - (1 - \xi) \frac{p_\mu p_\nu}{p^2 + i\varepsilon}\} , \quad (\text{A.5})$$

The case  $\xi = 1$  corresponds to the Feynman-'tHooft gauge,  $\xi = 0$  to the Landau gauge,  $\xi \rightarrow \infty$  to the Proca gauge.

As follows from (A.3) and (A.4), the propagators of massive vector fields can be written as sums of a propagator in the Proca gauge that describes the propagation of physical degrees of freedom of a vector particle, and a scalar propagator with a gauge-dependent pole which corresponds to the propagation of non-physical degrees of freedom. As a result, diagrams with virtual  $W^\pm$ ,  $Z$  bosons contain non-physical threshold singularities whose positions depend on the gauge parameter  $\xi$ . These non-physical singularities partially cancel out after the appropriate diagrams with virtual Goldstone bosons  $G^\pm$ ,  $G^0$  (arising from the higgs doublet  $\Phi$  (A.2)) are added. The Goldstone boson propagators have the form:

$$D_{G^+}(p) = \frac{i}{p^2 - m_W^2 \xi + i\varepsilon} \quad (\text{A.6})$$

$$D_{G^0}(p) = \frac{i}{p^2 - m_Z^2 \xi + i\varepsilon}. \quad (\text{A.7})$$

A complete restoration of unitarity (cancellation of non-physical singularities) and of gauge invariance (validity of the Ward identities) are achieved if one takes into account the diagrams with the Faddeev–Popov ghosts  $\eta^\pm$ ,  $\eta^Z$  and  $\eta^A$ , which interact only with gauge fields and the Goldstone fields and which do not correspond to any physical degrees of freedom.

Ghost propagators take the form

$$D_{\eta^+}(p) = \frac{i}{p^2 - m_W^2 \xi + i\varepsilon} , \quad (\text{A.8})$$

$$D_{\eta^Z}(p) = \frac{i}{p^2 - m_Z^2 \xi + i\varepsilon} , \quad (\text{A.9})$$

$$D_{\eta^A}(p) = \frac{i}{p^2 + i\varepsilon} \quad (\text{A.10})$$

Ghosts obey the Fermi statistics, so an additional sign  $(-1)$  must be ascribed to ghost loops, as one does for fermion loops.

The propagators of other fields are written as

$$\text{for the higgs field: } D_H(p) = \frac{i}{p^2 - m_H^2 + i\varepsilon} \quad (\text{A.11})$$

$$\text{for fermion fields: } \hat{D}_f(p) = \frac{i}{\hat{p} - m_f + i\varepsilon} \quad (\text{A.12})$$

In order to describe numerous three-particle vertices, it is convenient to unify the notations. Let us fix the momenta once and for all, as shown in figure 17, and let us denote a vertex by a set of fields in the following order: (ACB). The Feynman rules for three-particle vertices are then written as follows.

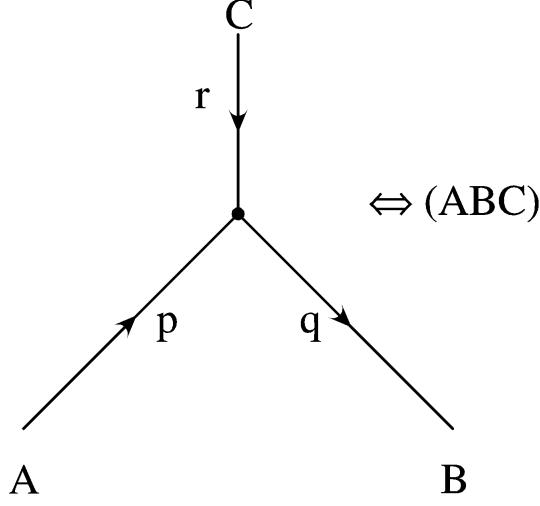


Figure 17: Three-particle vertex.

### Interaction between gauge fields and fermions.

$$\begin{aligned}
(f A_\mu f) &= -ieQ_f \gamma_\mu \quad , \\
(f Z_\mu f) &= -i\frac{f}{4}[g_V \gamma_\mu + g_A \gamma_\mu \gamma_5] \quad , \\
(\nu_e W_\mu^- l) &= -i\frac{g}{2\sqrt{2}} \gamma_\mu (1 + \gamma_5) \quad , \\
(U W_\mu^- D) &= -i\frac{g}{2\sqrt{2}} V_{DU} \gamma_\mu (1 + \gamma_5) \quad ;
\end{aligned} \tag{A.13}$$

where  $Q_f$  is the charge of the fermion  $f$ ,  $T_3^f$  is the third projection of the isotopic spinor, describing the left-handed component of the fermion  $f$ ,  $g_A = 2T_3^f$ ,  $g_V = 2T_3^f - 4Q_f \sin^2 \theta$ ;  $U$  and  $D$  denote any of the quarks with  $T_3^f = \frac{1}{2}$  and  $T_3^f = -\frac{1}{2}$ , respectively, and  $V_{DU}$  is an element of the Kobayashi–Maskawa matrix.

### Interaction of scalar fields with fermions.

$$\begin{aligned}
(f H f) &= -\frac{ig}{2m_W} m_f \quad , \\
(f G^0 f) &= -\frac{g}{4} \frac{m_f}{m_W} T_3^f \quad , \\
(UG^- D) &= -\frac{i}{2\sqrt{2}} \frac{g}{m_W} [(m_D - m_U) + \gamma_5(m_D + m_U)] \quad , \\
(DG^+ U) &= -\frac{i}{2\sqrt{2}} \frac{g}{m_W} [(m_D - m_U) - \gamma_5(m_D + m_U)] \quad .
\end{aligned} \tag{A.14}$$

### Three-boson interactions.

Three gauge bosons:

$$\begin{aligned}
(W_\lambda^+ A_\nu W_\mu^+) & \quad ie[r+q]_\lambda g_{\mu\nu} - (q+p)_\nu g_{\lambda\mu} + (p-r)_\mu g_{\nu\lambda} \quad , \\
(W_\lambda^+ Z_\nu W_\mu^+) & \quad ig \cos\theta [r+q]_\lambda g_{\mu\nu} - (q+p)_\nu g_{\lambda\mu} + (p-r)_\mu g_{\nu\lambda} \quad
\end{aligned} \tag{A.15}$$

Two gauge bosons and one scalar boson ( $G$  or  $H$ ):

$$\begin{aligned}
(W_\mu^+ G^- A_\nu) & \quad iem_W g_{\mu\nu} \quad , \\
(W_\mu^+ G^- Z_\nu) & \quad -igm_Z \sin^2\theta g_{\mu\nu} \quad , \\
(W_\mu^+ H W_\nu^+) & \quad igm_W g_{\mu\nu} \quad , \\
(Z_\mu H Z_\nu) & \quad ig \frac{m_Z^2}{m_W} g_{\mu\nu} \quad ;
\end{aligned} \tag{A.16}$$

One gauge boson and two scalar bosons ( $GG$  or  $GH$ ):

$$\begin{aligned}
(G^+ A_\mu G^+) & \quad -ie(p+q)_\mu \quad , \\
(G^+ Z_\mu G^+) & \quad -ig \frac{\cos 2\theta}{2\cos\theta} (p+q)_\mu \quad , \\
(G^0 W_\mu^+ G^+) & \quad \frac{1}{2}g(p+q)_\mu \quad , \\
(H W_\mu^+ G^+) & \quad -\frac{1}{2}ig(p+q)_\mu \quad , \\
(G^+ W_\mu^- G^0) & \quad -\frac{1}{2}g(p+q)_\mu \quad , \\
(G^+ W_\mu^- H) & \quad -\frac{1}{2}ig(p+q)_\mu \quad , \\
(H Z_\mu G^0) & \quad \frac{g}{\cos\theta} (p+q)_\mu \quad ;
\end{aligned} \tag{A.17}$$

Interaction of higgses with goldstones and among themselves:

$$\begin{aligned}
(G^+ H G^+) & \quad \frac{i}{2}g \frac{m_H^2}{m_W} \quad , \\
(G^- H G^-) & \quad -\frac{i}{2}g \frac{m_H^2}{m_W} \quad , \\
(G^0 H G^0) & \quad -\frac{i}{2}g \frac{m_H^2}{m_W} \quad , \\
(H H H) & \quad -\frac{3i}{2}g \frac{m_H^2}{m_W} \quad .
\end{aligned} \tag{A.18}$$

Interaction between ghosts and gauge fields:

$$\begin{aligned}
(\eta^+ A_\mu \eta^+) &= -ieq_\mu \ , \\
(\eta^- A_\mu \eta^-) &= ieq_\mu \ , \\
(\eta^- W_\mu^+ \eta^\gamma) &= -ieq_\mu \ , \\
(\eta^+ W_\mu^- \eta^\gamma) &= ieq_\mu \ , \\
(\eta^\gamma W_\mu^+ \eta^+) &= ieq_\mu \ , \\
(\eta^\gamma W_\mu^- \eta^-) &= -ieq_\mu \ , \\
(\eta^+ Z_\mu \eta^+) &= -ig \cos \theta \ q_\mu \ , \\
(\eta^- Z_\mu \eta^-) &= ig \cos \theta \ q_\mu \ ,
\end{aligned} \tag{A.19}$$

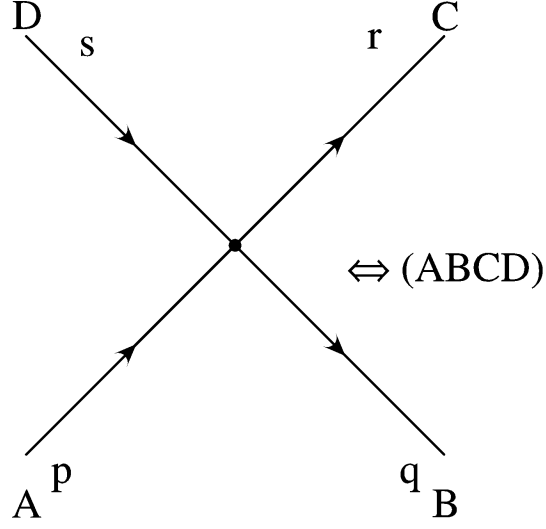


Figure 18: Four-particle vertex.

$$(\eta^- W_\mu^+ \eta^Z) = -ig \cos \theta q_\mu ,$$

$$(\eta^+ W_\mu^- \eta^Z) = ig \cos \theta q_\mu ,$$

$$(\eta^Z W_\mu^+ \eta^+) = ig \cos \theta q_\mu ,$$

$$(\eta^Z W_\mu^- \eta^-) = -ig \cos \theta q_\mu ;$$

Interaction of ghosts with a higgs or a goldstone:

$$(\eta^- H \eta^-) = -\frac{i}{2} g \xi m_W ,$$

$$(\eta^- G^0 \eta^-) = -\frac{1}{2} g \xi m_W ,$$

$$(\eta^+ G^0 \eta^+) = \frac{1}{2} g \xi m_W ,$$

$$(\eta^\gamma G^+ \eta^+) = -ie \xi m_W ,$$

$$(\eta^Z G^+ \eta^+) = -\frac{i}{2} g \frac{\cos 2\theta}{\cos \theta} \xi m_W ,$$

$$(\eta^- G^+ \eta^Z) = \frac{i}{2} g \xi m_Z .$$

(A.20)

#### Four-boson interactions.

To describe four-boson vertices, we introduce the notation (ABCD), see figure 18. In this

notation, the interactions of four vector bosons take the form

$$\begin{aligned}
(W_\lambda^+ W_\mu^- W_\nu^+ W_\rho^-) & \quad ig^2 [2g_{\lambda\nu} g_{\mu\rho} - g_{\mu\nu} g_{\lambda\rho} - g_{\mu\lambda} g_{\nu\rho}] \ , \\
(W_\lambda^+ W_\mu^- A_\nu A_\rho) & \quad -ie^2 [2g_{\nu\rho} g_{\mu\lambda} - g_{\mu\rho} g_{\nu\lambda} - g_{\mu\nu} g_{\lambda\rho}] \ , \\
(W_\lambda^+ W_\mu^- Z_\nu Z_\rho) & \quad -ig^2 \cos^2 \theta [2g_{\nu\rho} g_{\mu\lambda} - g_{\mu\rho} g_{\nu\lambda} - g_{\mu\nu} g_{\lambda\rho}] \ , \\
(W_\lambda^+ W_\mu^- A_\nu Z_\rho) & \quad -ieg \cos \theta [2g_{\nu\rho} g_{\mu\lambda} - g_{\mu\rho} g_{\nu\lambda} - g_{\mu\nu} g_{\lambda\rho}] \ .
\end{aligned} \tag{A.21}$$

Interaction between two vector bosons and  $HH$ ,  $GG$  or  $HG$ .

$$\begin{aligned}
(W_\mu^+ W_\nu^- HH) & \quad \frac{i}{2} g^2 g_{\mu\nu} \ , \\
(W_\mu^+ W_\nu^- G^0 G^0) & \quad \frac{i}{2} g^2 g_{\mu\nu} \ , \\
(W_\mu^+ W_\nu^- G^- G^+) & \quad \frac{i}{2} g^2 g_{\mu\nu} \ , \\
(A_\mu A_\nu G^- G^+) & \quad 2ie^2 g_{\mu\nu} \ . \\
\\ 
Z_\mu Z_\nu HH & \quad i \frac{1}{2} g^2 \sec^2 \theta g_{\mu\nu} \ , \\
Z_\mu Z_\nu G^0 G^0 & \quad i \frac{1}{2} g^2 \sec^2 \theta g_{\mu\nu} \ , \\
Z_\mu Z_\nu G^- G^+ & \quad i \frac{1}{2} g^2 \sec^2 \theta \cos^2 2\theta g_{\mu\nu} \ , \\
A_\mu W_\nu^+ G^+ H & \quad i \frac{1}{2} e g g_{\mu\nu} \ , \\
A_\mu W_\nu^+ G^- H & \quad i \frac{1}{2} e g g_{\mu\nu} \ , \\
\\ 
A_\mu W_\nu^+ G^+ G^0 & \quad -\frac{1}{2} e g g_{\mu\nu} \ , \\
A_\mu W_\nu^- G^- G^0 & \quad \frac{1}{2} e g g_{\mu\nu} \ , \\
A_\mu Z_\nu G^- G^+ & \quad ieg \sec \theta \cos 2\theta g_{\mu\nu} \ , \\
Z_\mu W_\nu^+ G^+ H & \quad i \frac{1}{2} g^2 \sec \theta \left( \frac{1}{2} \cos 2\theta - 1 \right) g_{\mu\nu} \ , \\
Z_\mu W_\nu^- G^- H & \quad i \frac{1}{2} g^2 \sec \theta \left( \frac{1}{2} \cos 2\theta - 1 \right) g_{\mu\nu} \ , \\
Z_\mu W_\nu^+ G^+ G^0 & \quad -\frac{1}{2} g^2 \sec \theta \left( \frac{1}{2} \cos 2\theta - 1 \right) g_{\mu\nu} \ , \\
Z_\mu W_\nu^- G^- G^0 & \quad \frac{1}{2} g^2 \sec \theta \left( \frac{1}{2} \cos 2\theta - 1 \right) g_{\mu\nu} \ .
\end{aligned} \tag{A.22}$$



Interactions of  $GGGG$ ,  $HHHH$  or  $GGHH$ .

$$\begin{aligned}
(G^+G^-G^-G^+) &= -\frac{i}{2}g^2m_H^2/m_W^2 \ , \\
(G^+G^-G^0G^0) &= -\frac{i}{4}g^2m_H^2/m_W^2 \ , \\
(G^+G^-HH) &= -\frac{i}{4}g^2m_H^2/m_W^2 \ , \\
(HHHH) &= -i\frac{3}{4}g^2m_H^2/m_W^2 \ , \\
(G^0G^0G^0G^0) &= -i\frac{3}{4}g^2m_H^2/m_W^2 \ , \\
(G^0G^0HH) &= -\frac{i}{4}g^2m_H^2/m_W^2 \ .
\end{aligned} \tag{A.23}$$

### Regularization of Feynman integrals.

Integrals corresponding to diagrams with loops formally diverge and thus need regularization. Note that there does not exist yet a consistent regularization of electroweak theory in all loops. A dimensional regularization can be used in the first several loops; this corresponds to a transition to a  $D$ -dimensional spacetime in which the following finite expression is assigned to the diverging integrals:

$$\int \frac{d^D p}{\mu^{D-4}} \frac{(p^2)^s}{(p^2 + m^2)^\alpha} = \frac{\pi^{\frac{D}{2}}}{\Gamma(\frac{D}{2})} \frac{\Gamma(\frac{D}{2} + s) \Gamma(\alpha - \frac{D}{2} - s)}{\Gamma(\alpha)} \times \frac{(m^2)^{\frac{D}{2} - \alpha + s}}{\mu^{D-4}} , \quad (\text{A.24})$$

where  $\mu$  is a parameter with mass dimension, introduced to conserve the dimension of the original integral.

This formula holds in the range of convergence of the integral. In the range of divergence, a formal expression (A.24) is interpreted as the analytical continuation. Obviously, the integral allows a shift in integration variable in the convergence range as well. Therefore, a shift  $p \rightarrow p + q$  for arbitrary  $D$  can also be done in (A.24). This factor is decisive in proving the gauge invariance of dimensional regularization.

At  $D = 4$  the integrals in (A.24) contain a pole term

$$\Delta = \frac{2}{4 - D} + \ln 4\pi - \gamma - \ln \frac{m^2}{\mu^2} , \quad (\text{A.25})$$

where  $\gamma = 0.577\dots$  is the Euler constant. Choice of constant terms in (A.25) is a matter of convention.

The algebra of  $\gamma$ -matrices in the  $D$ -dimensional space is defined by the relations

$$\gamma_\mu \gamma_\nu + \gamma_\nu \gamma_\mu = 2g_{\mu\nu} \times I , \quad (\text{A.26})$$

$$g_{\mu\mu} = D , \quad (\text{A.27})$$

$$\gamma_\mu \gamma_\nu \gamma_\mu = (2 - D) \gamma_\nu , \quad (\text{A.28})$$

where  $I$  is the identity matrix.

As for the dimensionality of spinors, different approaches can be chosen in the continuation to the  $D$ -dimensional space. One possibility is to assume that the  $\gamma$  matrices are  $4 \times 4$  matrices, so that

$$SpI = 4 . \quad (\text{A.29})$$

The  $D$ -dimensional regularization creates difficulties when one has to define the absolutely antisymmetric tensor and (or)  $\gamma_5$  matrix. For calculations in several first loops, a formal definition of  $\gamma_5$ ,

$$\gamma_5 \gamma_\mu + \gamma_\mu \gamma_5 = 0 , \quad (\text{A.30})$$

$$\gamma_5^2 = I \quad (\text{A.31})$$

does not lead to contradictions.

Thus, the amplitudes of physical processes, once they are expressed in terms of bare charges and bare masses, contain pole terms  $\sim 1/(D - 4)$ .

If we eliminate bare quantities and express some physical observables in terms of other physical observables, then all pole terms cancel out. The general property of renormalizability guarantees this cancellation. (We have verified this cancellation directly in [42].) This renormalization procedure is employed in this review.

In order to avoid divergences in intermediate expressions, one can agree to subtract from each Feynman integral the pole terms  $\sim 1/(4 - D)$ , since they will cancel out anyway in the final expressions. Depending on which constant terms (in addition to pole terms) are subtracted from the diagrams, different subtraction schemes arise: the  $\overline{MS}$  scheme corresponds to subtracting the universal combination

$$\frac{2}{4 - D} - \gamma + \ln 4\pi \quad .$$

## Appendix B.

### Relation between $\bar{\alpha}$ and $\alpha(0)$ .

We begin with the following famous relation of quantum electrodynamics [75]:

$$\alpha(q^2) = \frac{\alpha(0)}{1 + \Sigma_\gamma(q^2)/q^2 - \Sigma'_\gamma(0)} . \quad (\text{B.1})$$

Here the fine structure constant  $\alpha \equiv \alpha(0)$  is a physical quantity. It can be measured as a residue of the Coulomb pole  $1/q^2$  in the scattering amplitude of charged particles. As for the running coupling constant  $\alpha(q^2)$ , it can be measured from the scattering of particles with large masses  $m$  at low momentum transfer:  $m \gg \sqrt{|q^2|}$ . In the standard model we have the  $Z$ -boson, and the contribution of the photon cannot be identified unambiguously if  $q^2 \neq 0$ . Therefore, the definition of the running constant  $\alpha(q^2)$  becomes dependent on convention and on details of calculations.

At  $q^2 = m_Z^2$ , the contribution of  $W$ -bosons to  $\bar{\alpha} \equiv \alpha(m_Z^2)$  is not large, so it is convenient to make use of the definition accepted in QED:

$$\bar{\alpha} = \frac{\alpha}{1 - \delta\alpha} , \quad (\text{B.2})$$

where

$$\begin{aligned} \delta\alpha &= -\Pi_\gamma(m_Z^2) + \Sigma'_\gamma(0) , \\ \Pi_\gamma(m_Z^2) &= \frac{1}{m_Z^2} \Sigma_\gamma(m_Z^2) . \end{aligned} \quad (\text{B.3})$$

One-loop expression for the self-energy of the photon can be rewritten as [76]:

$$\begin{aligned} \Sigma_\gamma(s) &= (\alpha/3\pi) \sum_f N_c^f Q_f^2 [s\Delta_f + (s + 2m_f^2)F(s, m_f, m_f) - 3/2] - \\ &- (\alpha/4\pi) [3s\Delta_W + (3s + 4m_W^2)F(s, m_W, m_W)] , \end{aligned} \quad (\text{B.4})$$

where  $s \equiv q^2$ , the subscript  $f$  denotes fermions, the sum  $\Sigma_f$  runs through lepton and quark flavors, and  $N_c^f$  is the number of colors. The contribution of fermions to  $\Sigma_\gamma(q^2)$  is independent of gauge. The last term in (B.4) refers to the gauge-dependent contribution of  $W$ -bosons; the 't Hooft–Feynman gauge was used in equation (B.4).

The singular term  $\Delta_i$  is:

$$\Delta_i = \frac{1}{\epsilon} - \gamma + \ln 4\pi - \ln \frac{m_i^2}{\mu^2} , \quad (\text{B.5})$$

where  $2\epsilon = 4 - D$  ( $D$  is the variable dimension of spacetime,  $\epsilon \rightarrow 0$ ),  $\gamma = -\Gamma'(1) = 0.577\dots$  is the Euler constant and  $\mu$  is an arbitrary parameter. Both  $1/\epsilon$  and  $\mu$  vanish in relations between observables.

The function  $F(s, m_1, m_2)$  is defined by the contribution to self-energy of a scalar particle at  $q^2 = s$ , owing to a loop with two scalar particles (with masses  $m_1$  and  $m_2$ ) and with the coupling constant equal to unity:

$$F(s, m_1, m_2) = -1 + \frac{m_1^2 - m_2^2}{m_1^2 - m_2^2} \log \frac{m_1}{m_2} - \int_0^1 dx \log \frac{x^2 s - x(s + m_1^2 - m_2^2) + m_1^2 - i\epsilon}{m_1 m_2} . \quad (\text{B.6})$$

The function  $F$  is normalized in such a way that it vanishes at  $q^2 = 0$ , which corresponds to subtracting the self-energy at  $q^2 = 0$ :

$$F(0, m_1, m_2) = 0 , \quad (\text{B.7})$$

The following formula holds for  $m_1 = m_2 = m$ :

$$F(s, m, m) \equiv F(\tau) = \begin{cases} 2 \left[ 1 - \sqrt{4\tau - 1} \arcsin \frac{1}{\sqrt{4\tau}} \right] , & 4\tau > 1 , \\ 2 \left[ 1 - \sqrt{1 - 4\tau} \ln \frac{1 + \sqrt{1 - 4\tau}}{\sqrt{4\tau}} \right] , & 4\tau < 1 , \end{cases}$$

where  $\tau = m^2/s$ .

To calculate the contributions of light fermions, the  $t$ -quark and the  $W$ -boson to  $\delta\alpha$ , we need the asymptotics  $F(\tau)$  for small and large  $\tau$ :

$$F(\tau) \simeq \ln \tau + 2 + \dots, \quad |\tau| \ll 1 , \quad (\text{B.8})$$

$$F(\tau) \simeq \frac{1}{6\tau} + \frac{1}{60\tau^2} + \dots, \quad |\tau| \gg 1 , \quad (\text{B.9})$$

$$F'(s, m, m) = \frac{d}{ds} F(s, m, m) \stackrel{s \rightarrow 0}{\simeq} \frac{1}{m^2} \left[ \frac{1}{6} + \frac{1}{30\tau} \right] . \quad (\text{B.10})$$

As a result we obtain

$$\begin{aligned} \Pi_\gamma(m_Z^2) &\equiv \frac{\Sigma_\gamma(m_Z^2)}{m_Z^2} = \frac{\alpha}{3\pi} \sum_8 N_c^f Q_f^2 (\Delta_Z + \frac{5}{3}) + \\ &+ \frac{\alpha}{\pi} Q_f^2 \left[ \Delta_t + (1 + 2t)F(t) - \frac{1}{3} \right] - \\ &- \frac{\alpha}{4\pi} [3\Delta_W + (3 + 4c^2)F(c^2)] , \end{aligned} \quad (\text{B.11})$$

where  $t = m_t^2/m_Z^2$ , and

$$\Sigma'_\gamma(0) = \frac{\alpha}{3\pi} \sum_9 N_c^f Q_f^2 \Delta_f - \frac{\alpha}{4\pi} (3\Delta_W + \frac{2}{3}) , \quad (\text{B.12})$$

$$\begin{aligned} \delta\alpha &= \frac{\alpha}{\pi} \left\{ \sum_8 \frac{N_c^f Q_f^2}{3} \left( \ln \frac{m_Z^2}{m_f^2} - \frac{5}{3} \right) - \right. \\ &- Q_t^2 \left[ (1 + 2t)F(t) - \frac{1}{3} \right] + \left[ \left( \frac{3}{4} + c^2 \right) F(c^2) - \frac{1}{6} \right] \left. \right\} . \end{aligned} \quad (\text{B.13})$$

Therefore,  $\delta\alpha$  is found as a sum of four terms,

$$\delta\alpha = \delta\alpha_l + \delta\alpha_h + \delta\alpha_t + \delta\alpha_W \quad , \quad (\text{B.14})$$

$$\delta\alpha_l = \frac{\alpha}{3\pi} \sum_3 \left[ \ln \frac{m_Z^2}{m_t^2} - 5/3 \right] = 0.03141 \quad . \quad (\text{B.15})$$

$$\delta\alpha_t \simeq -\frac{\alpha}{\pi} \frac{4}{45} \left( \frac{m_Z}{M_t} \right)^2 = -0.00005(1) \quad , \quad (\text{B.16})$$

where we have used that  $m_t = 175 \pm 10$  GeV. Note that  $\delta\alpha_t$  is negligible and has the antiscreening sign (the screening of the  $t$ -quark loops in QED begins at  $q^2 \gg m_t^2$ , while in our case  $q^2 = m_Z^2 < m_t^2$ ).

Finally, the  $W$  loop gives

$$\delta\alpha_W = \frac{\alpha}{2\pi} \left[ (3 + 4c^2) \left( 1 - \sqrt{4c^2 - 1} \arcsin \frac{1}{2c} \right) - 1/3 \right] = 0.00050 \quad . \quad (\text{B.17})$$

The value of  $\delta\alpha_W$  depends on gauge [77]; here we give the result of calculations in the 't Hooft–Feynman gauge. Traditionally, the definition of  $\bar{\alpha}$  takes into account the contributions of leptons and five light quarks; the terms  $\delta\alpha_t$  and  $\delta\alpha_W$  are taken into account in the electroweak radiative corrections. In our approach, these terms give the corrections  $\delta_1 V_i$ .

## Appendix C

### Summary of the results for $\bar{\alpha}$ .

Among the three input parameters  $\bar{\alpha}$ ,  $G_\mu$  and  $M_Z$ , the first one has the maximum uncertainty; this uncertainty leads to the uncertainty  $\pm 5$  GeV in the value of the  $t$ -quark mass extracted from the measurements of  $m_W$  and the decay parameters of the  $Z$ -boson. According to the formulas of Appendix B,

$$\bar{\alpha} = \frac{\alpha}{1 - \delta\alpha} \quad , \delta\alpha = \delta\alpha_l + \delta\alpha_h \quad ,$$

$$\delta\alpha_l = 0.0314 \quad .$$

In ([26], Burkh)  $\delta\alpha_h$  was calculated by substituting experimental data on  $\sigma_{e^+e^- \rightarrow \text{hadrons}}$  into the dispersion integral at  $\sqrt{s} < 40$  GeV and the parton model result at  $\sqrt{s} > 40$  GeV:

$$\delta\alpha_h^{[\text{Burkh}]} = 0.0282(9) \quad , \quad \bar{\alpha}^{[\text{Burkh}]} = [128.87(12)]^{-1} \quad . \quad (\text{C.1})$$

In ([26], Vys) it was pointed out that the simplest model (the lightest vector meson + the QCD-improved parton continuum in each flavor channel) produces a surprisingly close result:

$$\delta\alpha_h^{[\text{Vys}]} = 0.0280(4) \quad , \quad \bar{\alpha}^{[\text{Vys}]} = [128.90(6)]^{-1} \quad . \quad (\text{C.2})$$

The model with infinite number of poles ([26], Gesh) yields the determination of  $\delta\alpha_h$  with very high accuracy:

$$\delta\alpha_h^{[\text{Gesh}]} = 0.0275(2) \quad , \quad \bar{\alpha}^{[\text{Gesh}]} = [128.96(3)]^{-1} \quad . \quad (\text{C.3})$$

A recent analysis of experimental data ([26], Sw, [26], Ma) yielded considerably lower values of  $\delta\alpha_h$ :

$$\delta\alpha_h^{[\text{Sw}]} = 0.0265(8) \quad , \quad \bar{\alpha}^{[\text{Sw}]} = [129.10(12)]^{-1} \quad , \quad (\text{C.4})$$

$$\delta\alpha_h^{[\text{Ma}]} = 0.0273(4) \quad , \quad \bar{\alpha}^{[\text{Ma}]} = [128.99(6)]^{-1} \quad . \quad (\text{C.5})$$

In this review we make use of the results of a recent analysis ([26], Eid),

$$\delta\alpha_h^{[\text{Eid}]} = 0.0280(7) \quad , \quad \bar{\alpha}^{[\text{Eid}]} = [128.896(90)] \quad . \quad (\text{C.6})$$

## Appendix D.

### How $\alpha_W(q^2)$ and $\alpha_Z(q^2)$ ‘crawl’.

The effect of ‘running’ of electromagnetic coupling constants  $\alpha(q^2)$  (logarithmic dependence of the effective charge on momentum transfer  $q^2$ ) is known for more than four decades [75]. In contrast to  $\alpha(q^2)$ , the effective constants of  $W$  and  $Z$  bosons  $\alpha_W(q^2)$  and  $\alpha_Z(q^2)$  in the region  $0 < q^2 \lesssim m_Z^2$  ‘crawl’ rather than run [78].

If we define the effective gauge coupling constant  $g^2(q^2)$  in terms of the bare charge  $g_0^2$  and the bare mass  $m_0$ , and sum up the geometric series with the self-energy  $\Sigma(q^2)$  inserted in the gauge boson propagator, this gives the expression

$$g^2(q^2) = \frac{g_0^2}{1 + g_0^2 \frac{\Sigma(q^2) - \Sigma(m^2)}{q^2 - m^2}} . \quad (\text{D.1})$$

Here  $m$  is the physical mass, and  $\Sigma(q^2)$  contains the contribution of fermions only, since loops with  $W$ ,  $Z$  and  $H$  bosons do not contain large logarithms in the region  $|q^2| \leq m_Z^2$ .

The bare coupling constant in the difference  $1/g^2(q^2) - 1/g^2(0)$  is eliminated, which gives a finite expression. The result is

$$1/\alpha_Z(q^2) - 1/\alpha_Z(0) = b_Z F(x) , \quad \text{where } x = q^2/m_Z^2 , \quad (\text{D.2})$$

$$1/\alpha_W(q^2) - 1/\alpha_W(0) = b_W F(y) , \quad \text{where } y = q^2/m_Z^2 , \quad (\text{D.3})$$

$$F(x) = \frac{x}{1-x} \ln |x| \quad (\text{D.4})$$

If  $x \gg 1$ , equations (D.2) and (D.3) define the logarithmic running of charges owing to leptons and quarks, and  $b_Z$  and  $b_W$  represent the contribution of fermions to the first coefficient of the Gell-Mann–Low function:

$$\begin{aligned} b_Z &= \frac{1}{48\pi} \{ N_u 3[1 + (1 - \frac{8}{3}s^2)^2] + N_d 3[1 + (-1 + \frac{4}{3}s^2)^2] + N_l [2 + (1 + (1 - 4s^2)^2)] \} , \\ b_W &= \frac{1}{16\pi} \{ 6N_q + 2N_l \} , \end{aligned} \quad (\text{D.5})$$

where  $N_{u,d,q,l}$  are the numbers of quarks and leptons with masses that are considerably lower than  $\sqrt{q^2}$ .

For  $q^2 \lesssim m_Z^2$ , the numerical values of the coefficients  $b_{Z,W}$  are [78]

$$b_Z \simeq 0.195$$

$$b_W \simeq 0.239$$

The massive propagator  $\frac{1}{q^2 - m^2}$  in (D.1) greatly suppresses the running of  $\alpha_W(q^2)$  and  $\alpha_Z(q^2)$ . Thus, according to (D.2) and (D.3), the constant  $\alpha_Z(q^2)$  grows by 0.85% from  $q^2 = 0$  to  $q^2 = m_Z^2$ ,

$$\begin{aligned} 1/\alpha_Z(m_Z^2) &= 22.905 \\ 1/\alpha_Z(m_Z^2) - 1/\alpha_Z(0) &= -0.195 , \end{aligned} \quad (\text{D.6})$$



and the constant  $\alpha_W(q^2)$  grows by 0.95%,

$$1/\alpha_W(m_Z^2) = 28.74$$

$$1/\alpha_W(m_Z^2) - 1/\alpha_W(0) = -0.272 \quad , \quad (\text{D.7})$$

while the electromagnetic constant  $\alpha(q^2)$  increases by 6.34%:

$$1/\alpha(m_Z^2) - 1/\alpha_W(0) = 128.90 - 137.04 = -8.14 \quad (\text{D.8})$$

With the accuracy indicated above, we can thus assume

$$\begin{aligned} \alpha_Z(m_Z^2) &\simeq \alpha_Z(0) \\ \alpha_W(m_Z^2) &\simeq \alpha_W(0). \end{aligned} \quad (\text{D.9})$$

At the same time,  $\alpha(m_Z^2)$  differs greatly from  $\alpha(0)$ ; therefore the latter has no connection to the electroweak physics but only to the purely electromagnetic physics.

## Appendix E

### Relation between $\bar{\alpha}$ , $G_\mu$ , $m_Z$ and the bare quantities.

The bare quantities are marked by the subscript ‘0’. In the electroweak theory, three bare charges  $e_0$ ,  $f_0$  and  $g_0$  that describe the interactions of  $\gamma$ ,  $Z$  and  $W$  are related by a single constraint:

$$(e_0/g_0)^2 + (g_0/f_0)^2 = 1 . \quad (\text{E.1})$$

The bare masses of the vector bosons are defined by the bare vacuum expectation value of the higgs field  $\eta_0$ :

$$m_{Z0} = \frac{1}{2}f_0\eta_0 , \quad m_{W0} = \frac{1}{2}g_0\eta_0 . \quad (\text{E.2})$$

The fine structure constant  $\alpha = e^2/4\pi$  is related to the bare charge  $e_0$  by the formula

$$\alpha \equiv \alpha(q^2 = 0) = \frac{e_0^2}{4\pi} \left( 1 - \Sigma'_\gamma(0) - 2\frac{s}{c} \frac{\Sigma_{\gamma Z}(0)}{m_Z^2} \right) , \quad (\text{E.3})$$

where  $\Sigma'(0) = \lim_{q^2 \rightarrow 0} \Sigma(q^2)/q^2$ . In the Feynman gauge  $\Sigma_{\gamma Z}(0) \approx -(\alpha/2\pi)(m_W^2/cs)(1/\epsilon)$ , where the dimension of spacetime is  $D = 4 - 2\epsilon$ . In the unitary gauge  $\Sigma_{\gamma Z}(0) = 0$ .

The simplest way to verify the presence of the term  $2(s/c)\Sigma_{\gamma Z}(0)/m_Z^2$  is by considering the interaction of a photon with the right-handed electron  $e_R$ . Note that in this case there are no weak vertex corrections due to the  $W$ -boson exchange. (Note also that the left-handed neutrino remains neutral even when loop corrections are taken into account, since the diagram with the  $\gamma - Z - \nu_L \bar{\nu}_L$  interaction is compensated for by the vertex diagram with the  $W$  exchange).

Our first basic equation is the renormalization group improved relation between  $\bar{\alpha} = \alpha(q^2 = m_Z^2)$  and  $\alpha_0$ :

$$\bar{\alpha} = \alpha_0 \left[ 1 - \Pi_\gamma(m_Z^2) - 2\frac{s}{c}\Pi_{\gamma Z}(0) \right] , \quad (\text{E.4})$$

where  $\Pi_\gamma(q^2) = \Sigma_\gamma(q^2)/m_Z^2$ ,  $\Pi_{\gamma Z}(q^2) = \Sigma_{\gamma Z}(q^2)/m_Z^2$ .

The second basic equation is:

$$m_Z^2 = m_{Z0}^2[1 - \Pi_Z(m_Z^2)] = m_{W0}^2/c_0^2[1 - \Pi_Z(m_Z^2)] . \quad (\text{E.5})$$

A similar equation holds for  $m_W^2$ :

$$m_W^2 = m_{W0}^2[1 - \Pi_W(m_W^2)] , \quad (\text{E.6})$$

where  $\Pi_i(q^2) = \Sigma_i(q^2)/m_i^2$ ,  $i = W, Z$ .

Finally, the third basic equation is

$$G_\mu = \frac{g_0^2}{4\sqrt{2}m_{W0}^2} [1 + \Pi_W(0) + D] , \quad (\text{E.7})$$

where  $\Pi_W(0) = \Sigma_W(0)/m_W^2$  comes from the propagator of  $W$ , while  $D$  is the contribution of the box and the vertex diagrams (minus the electromagnetic corrections to the four-fermion interaction) to the muon decay amplitude. According to Sirlin [23],

$$D = \frac{\bar{\alpha}}{4\pi s^2} \left( 6 + \frac{7 - 4s^2}{2s^2} \ln c^2 + 4\Delta_W \right) , \quad (\text{E.8})$$

where

$$\Delta_W \equiv \Delta(m_W) = \frac{2}{4-D} + \ln 4\pi - \gamma - \ln(m_W^2/\mu^2) \quad (\text{E.9})$$

## Appendix F

### The radiators $R_{Aq}$ and $R_{Vq}$ .

For decays to light quarks  $q = u, d, s$ , we neglect the quark masses and take into account the gluon exchanges in the final state up to terms  $\sim \alpha_s^3$  [45] - [48], and also one-photon exchange in the final state and the interference of the photon and the gluon exchanges [49]. These corrections are slightly different for the vector and the axial channels.

For decays to quarks we have

$$\Gamma_q = \Gamma(Z \rightarrow q\bar{q}) = 12[g_{Aq}^2 R_{Aq} + g_{Vq}^2 R_{Vq}] \Gamma_0 \quad (\text{F.1})$$

where the factors  $R_{A,V}$  are responsible for the interaction in the final state (in our previous papers, we used letter  $G$  instead of  $R$ ). The results presented in [45] - [49] are

$$\begin{aligned} R_{Vq} = & 1 + \frac{\hat{\alpha}_s}{\pi} + \frac{3}{4} Q_q^2 \frac{\bar{\alpha}}{\pi} - \frac{1}{4} Q_q^2 \frac{\bar{\alpha}}{\pi} \frac{\hat{\alpha}_s}{\pi} + \\ & + [1.409 + (0.065 + 0.015 \ln t) \frac{1}{t}] (\frac{\hat{\alpha}_s}{\pi})^2 - 12.77 (\frac{\hat{\alpha}_s}{\pi})^3 + 12 \frac{\hat{m}_q^2}{m_Z^2} \frac{\hat{\alpha}_s}{\pi} \delta_{vm} \end{aligned} \quad (\text{F.2})$$

$$\begin{aligned} R_{Aq} = & R_{Vq} - (2T_{3q}) [I_2(t) (\frac{\hat{\alpha}_s}{\pi})^2 + I_3(t) (\frac{\hat{\alpha}_s}{\pi})^3] - \\ & - 12 \frac{\hat{m}_q^2}{m_Z^2} \frac{\hat{\alpha}_s}{\pi} \delta_{vm} - 6 \frac{\hat{m}_q^2}{m_Z^2} \delta_{am}^1 - 10 \frac{\hat{m}_q^2}{m_t^2} (\frac{\hat{\alpha}_s}{\pi})^2 \delta_{am}^2 \end{aligned} \quad (\text{F.3})$$

where  $\hat{m}_q$  is the running quark mass (see below),

$$\delta_{vm} = 1 + 8.7 (\frac{\hat{\alpha}_s}{\pi}) + 45.15 (\frac{\hat{\alpha}_s}{\pi})^2, \quad (\text{F.4})$$

$$\delta_{am}^1 = 1 + 3.67 (\frac{\hat{\alpha}_s}{\pi}) + (11.29 - \ln t) (\frac{\hat{\alpha}_s}{\pi})^2, \quad (\text{F.5})$$

$$\delta_{am}^2 = \frac{8}{81} + \frac{\ln t}{54}, \quad (\text{F.6})$$

$$I_2(t) = -3.083 - \ln t + \frac{0.086}{t} + \frac{0.013}{t^2}, \quad (\text{F.7})$$

$$\begin{aligned} I_3(t) &= -15.988 - 3.722 \ln t + 1.917 \ln^2 t, \\ t &= m_t^2 / m_Z^2. \end{aligned} \quad (\text{F.8})$$

Terms of the order of  $(\frac{\hat{\alpha}_s}{\pi})^3$  caused by the diagrams with three gluons in intermediate state were calculated in [79]. For  $R_{Vq}$  they are numerically very small,  $\sim 10^{-5}$ ; for this reason, we dropped them from formula (F.2).

For the  $Z \rightarrow b\bar{b}$  decay, the  $b$ -quark mass is not negligible; it reduces  $\Gamma_b$  by about 1 MeV ( $\sim 0.5\%$ ). The gluon corrections result in a replacement of the pole mass  $m_b \simeq 4.7\text{GeV}$  by the running mass, the virtuality being  $m_Z$  :  $m_b \rightarrow \hat{m}_b(m_Z)$ . We express  $\hat{m}_b(m_Z)$  in terms of  $m_b$ ,  $\hat{\alpha}_s(m_Z)$  and  $\hat{\alpha}_s(m_b)$  using standard two-loop equations in the  $\overline{MS}$  scheme (see [50]).

For the  $Z \rightarrow c\bar{c}$  decay, the running mass  $\hat{m}_c(m_Z)$  is of the order of 0.5 GeV and the corresponding contribution to  $\Gamma_c$  is of the order of 0.05 MeV. We have included this infinitesimal term in the LEPTOP code, since it is taken into account in other codes (see, for example, [18]).

We need to remark in connection with  $\Gamma_c$  that the term  $I_2(t)$ , given by equation (F.7), contains interference terms  $\sim (\hat{\alpha}_s/\pi)^2$ . These terms are related to three types of final states: one quark pair, a quark pair and a gluon, two quark pairs. This last contribution comes to about 5% of  $I_2$  and is infinitesimally small at the currently achievable experimental accuracy. Nevertheless, in principle these terms require special consideration, especially if these quark pairs are of different flavors, for example,  $b\bar{b}c\bar{c}$ . Such mixed quark pairs must be discussed separately.

Note that  $\hat{\alpha}_s$  stands for the strong interaction constant in the  $\overline{MS}$  subtraction scheme, with  $\mu^2 = m_Z^2$ .

## Appendix G.

### Derivation of formulas for the asymmetries.

Asymmetry in the processes  $e^+e^- \rightarrow Z \rightarrow f\bar{f}$  is calculated with the masses of  $e$  and  $f$  neglected in comparison with the  $Z$  boson mass. (Mass corrections for  $f = b$  are of the order of  $2 \cdot 10^{-3}$  and will be taken into account below). The amplitude (20) of the interactions of the  $Z$  boson with massless fermions  $f\bar{f}$  can be conveniently rewritten in the form

$$M(Z \rightarrow f\bar{f}) = \frac{1}{2}\bar{f}Z_\alpha[g_L^f j_\alpha^L + g_R^f j_\alpha^R] , \quad (\text{G.1})$$

where

$$\begin{aligned} j_\alpha^{L,R} &= \bar{\psi}_{L,R}\gamma_\alpha\psi_{L,R} , \\ \psi_{L,R} &= \frac{1}{2}(1 \pm \gamma_5)\psi , \\ g_{L,R}^f &= g_{Vf} \pm g_{Af} . \end{aligned}$$

The chirality is a conserved quantum number for massless fermions (anomalies do not yet manifest themselves in the approximations we deal with here) and coincides with a fermion's helicity up to the sign.

Therefore, the pairs  $e_L^-e_L^+$  and  $e_R^-e_R^+$  do not transform into the  $Z$  boson at all, and the pairs  $e_L^-e_R^+$  and  $e_R^-e_L^+$  create a  $Z$  boson with the polarization  $\pm 1$ , respectively (along the positron beam). The scattering amplitudes thus have the form

$$\begin{aligned} T(e_{L,R}^-e^+ \rightarrow f_{L,R}\bar{f}) &= g_{L,R}^e g_{L,R}^f T_0(1 + \cos\theta) \\ T(e_{L,R}^-e^+ \rightarrow f_{R,L}\bar{f}) &= g_{L,R}^e g_{R,L}^f T_0(1 - \cos\theta) , \end{aligned} \quad (\text{G.2})$$

where coefficient  $T_0$  is nonimportant at the moment (it can be reconstructed from (G.1)), and  $\theta$  is the angle between the momenta of  $e^-$  and  $f$ . The sign in front of  $\cos\theta$  is chosen for the helicity to be conserved in forward and backward scattering.

Once the form of the amplitude (G.2) is known, all asymmetries are immediately found.

(a) Left-right asymmetry  $A_{LR}$  is defined as the ratio

$$A_{LR} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} ,$$

where  $\sigma_{L,R} = \sigma(e_{L,R}e^+ \rightarrow f\bar{f})$ . Hence

$$A_{LR} = \frac{(g_L^e)^2 - (g_R^e)^2}{(g_L^e)^2 + (g_R^e)^2} \equiv A^e \quad (\text{G.3})$$

(b) Longitudinal polarization  $P_\tau(\cos\theta)$  is defined as the ratio of the difference to the sum of differential cross sections,  $(\frac{d\sigma}{d\theta})_{R,L} = \frac{d\sigma}{d\theta}(e\bar{e} \rightarrow \tau_{R,L}\bar{\tau})$ :

$$P_\tau(\cos\theta) = \frac{(\frac{d\sigma}{d\theta})_R - (\frac{d\sigma}{d\theta})_L}{(\frac{d\sigma}{d\theta})_R + (\frac{d\sigma}{d\theta})_L} , \quad (\text{G.4})$$

where

$$\begin{aligned} \left(\frac{d\sigma}{d\theta}\right)_R &= \frac{1}{2m_Z^2} |T_0|^2 (g_R^\tau)^2 [(g_R^e)^2 (1 + \cos \theta)^2 + (g_L^e)^2 (1 - \cos \theta)^2] \\ \left(\frac{d\sigma}{d\theta}\right)_L &= \frac{1}{2m_Z^2} |T_0|^2 (g_L^\tau)^2 [(g_L^e)^2 (1 + \cos \theta)^2 + (g_R^e)^2 (1 - \cos \theta)^2] \end{aligned} \quad (\text{G.5})$$

Substituting (G.5) into the definition (G.4), we obtain

$$P_\tau(\cos \theta) = -\frac{A_\tau(1 + \cos^2 \theta) + 2A_e \cos \theta}{(1 + \cos^2 \theta) + 2A_e A_\tau \cos \theta}, \quad (\text{G.6})$$

where  $A_e$  and  $A_\tau$  are defined according to (G.3). The longitudinal polarization  $P^\tau$ , averaged over directions of  $\tau$ -leptons, is defined as the following ratio of the total cross sections  $\sigma_{L,R} = \int_{-1}^1 d\cos \theta \left(\frac{d\sigma}{d\theta}\right)_{L,R}$

$$\begin{aligned} P_\tau &= \frac{\sigma_R^\tau - \sigma_L^\tau}{\sigma_R^\tau + \sigma_L^\tau} = -\frac{\int_{-1}^1 d\cos \theta [A_\tau(1 + \cos^2 \theta) + 2A_e \cos \theta]}{\int_{-1}^1 d\cos \theta [1 + \cos^2 \theta + 2A_e A_\tau \cos \theta]} \\ &= -A_\tau \end{aligned} \quad (\text{G.7})$$

(c) Forward-backward asymmetry  $A_{FB}^f$  is calculated more simply in terms of  $g_{A,V}$ . The squared matrix element of the process  $e\bar{e} \rightarrow f\bar{f}$  is proportional to

$$\begin{aligned} |M|^2 &\sim \left\{ [g_{Ae}^2 + g_{Ve}^2] [g_{Af}^2 + g_{Vf}^2] (1 + v^2 \cos^2 \theta) \right. \\ &\quad \left. + [g_{Af}^2 - g_{Vf}^2] (1 - v^2) \right\} + \frac{1}{2} (g_{Ve} g_{Ae} g_{Vf} g_{Af}) v \cos \theta \quad , \end{aligned} \quad (\text{G.8})$$

where  $\theta$  is the scattering angle and  $v = 1 - \frac{4m_f^2}{m_Z^2}$  is the velocity of fermion  $f$ . This immediately implies that

$$A_{FB}^f = \frac{3}{4} A_e \left[ \frac{2g_{Af} g_{Vf} v}{g_{Af}^2 v^2 + g_{Vf}^2 \frac{3-v^2}{2}} \right] \quad (\text{G.9})$$

The mass  $m_f$  is negligible in all channels with the exception of  $f = b$ , where the nonzero mass produces effects of the order of  $2 \cdot 10^{-3}$ . Gluon corrections in the final state (see Appendix F) replace the pole mass  $m_b \simeq 4.7$  GeV in equations (G.8)–(G.9) by the running mass at the  $m_Z$  scale:  $m_b \rightarrow \hat{m}_b(m_Z)$ .

Note that starting from gluon corrections of the order of  $(\frac{\alpha_s}{\pi})^2$ , it is impossible to unambiguously separate different quark channels, since additional pairs of ‘alien’ quarks are created in this order. We do not consider corrections  $(\frac{\hat{\alpha}_s}{\pi})^2$  in asymmetries. In our approximation the ratio  $g_{Vf}/g_{Af}$  is not renormalized by the gluonic interaction in the final state. Therefore, the expected accuracy of formula (G.9) is  $(\frac{\alpha_s}{\pi})^2 \sim 2 \cdot 10^{-3}$ , which is by an order of magnitude better than the experimental accuracy.

## Appendix H.

### Corrections proportional to $m_t^2$ .

This appendix gives a simple mnemonic recipe for the derivation of corrections proportional to  $m_t^2$ . A rigorous derivation requires careful regularization of Feynman integrals.

The terms proportional to  $m_t^2$  contribute to radiative corrections to bare masses (squared) of the  $W$ - and  $Z$ -bosons, but not to the corrections to the bare coupling constants. This follows from dimensional arguments. Indeed, the dimension of self-energy  $\Sigma$  for the boson, equals  $m^2$ ; therefore, the terms  $\sim m_t^2$  remain in  $\Sigma(q^2)$  in the limit  $q^2 \rightarrow 0$ . On the other hand, the corrections to coupling constants are proportional to  $d\Sigma/dq^2$  and do not contain terms  $\sim m_t^2$ . Therefore, it is easy to evaluate the contribution of the  $t$ -quark to the parameter  $\rho = (\alpha_Z/\alpha_W)(m_W^2/m_Z^2)$  in the approximation  $\sim \alpha m_t^2$  (the Veltman approximation [29]), neglecting the terms  $\sim \alpha$ :

$$\begin{aligned} \rho &\simeq \frac{\alpha_{Z0}}{\alpha_{W0}} \frac{m_{W0}^2 - \Sigma_W(m_W^2)}{m_{Z0}^2 - \Sigma_Z(m_Z^2)} \simeq 1 + \frac{\Sigma_Z(0)}{m_Z^2} - \frac{\Sigma_W(0)}{m_W^2} \equiv \\ &\equiv 1 + \Pi_Z(0) - \Pi_W(0) . \end{aligned} \quad (\text{H.1})$$

The evaluation of the difference  $\Pi_Z(0) - \Pi_W(0)$  is elementary:

$$\begin{aligned} \Pi_W(0) &= \frac{\Sigma_W(0)}{m_W^2} = \\ &= \frac{3\alpha_W}{8\pi m_W^2} \left( \int_0^\infty \frac{p^2 dp^2}{p^2 + m_t^2} = \int_0^\infty dp^2 - m_t^2 \int_0^\infty \frac{dp^2}{p^2 + m_t^2} \right) . \end{aligned} \quad (\text{H.2})$$

(As we have neglected the mass of the  $b$ -quark, the propagator of the  $b$ -quark compensates the factor  $p^2$  in the numerator.)

$$\begin{aligned} \Pi_Z(0) &= \frac{\Sigma_Z(0)}{m_Z^2} = \\ &= \frac{3\alpha_Z}{8\pi m_Z^2} \left( \frac{1}{2} \int_0^\infty dp^2 + \frac{1}{2} \int_0^\infty \frac{p^4 dp^2}{(p^2 + m_t^2)^2} = \right. \\ &= \left. \frac{1}{2} \int_0^\infty dp^2 + \frac{1}{2} \int_0^\infty -m_t^2 \int_0^\infty \frac{dp^2}{p^2 + m_t^2} + \frac{1}{2} m_t^2 \right) . \end{aligned} \quad (\text{H.3})$$

Taking into account that in one-loop approximation we can set in (H.2) and (H.3)

$$\frac{\alpha_W}{m_W^2} = \frac{\alpha_Z}{m_Z^2} , \quad (\text{H.4})$$

we see that quadratic and logarithmic divergences cancel out and that finally

$$\rho \simeq 1 + \Delta\rho_t = 1 + \frac{3\alpha_Z}{16\pi} \frac{m_t^2}{m_Z^2} = 1 + \frac{3\alpha_Z}{16\pi} t , \quad (\text{H.5})$$

$$\Delta\rho_t = \frac{3\bar{\alpha}}{16\pi s^2 c^2} t , \quad (\text{H.6})$$



where  $t = m_t^2/m_Z^2$  and we assume that  $t \gg 1$ .

Let us express the leading in  $t$  corrections to the main quantities  $m_W/m_Z$ ,  $g_{A_i}$  and  $g_{V_i}$  in terms of  $\Delta\rho_t$ .

We define

$$c_\alpha^2 = \alpha_W/\alpha_Z, s_\alpha^2 = 1 - c_\alpha^2. \quad (\text{H.7})$$

Then

$$G_\mu = \frac{\pi\alpha_W}{\sqrt{2}m_W^2} = \frac{\pi}{\sqrt{2}\rho} \frac{\bar{\alpha}}{c_\alpha^2 s_\alpha^2 m_Z^2}, \quad (\text{H.8})$$

and hence

$$s_\alpha^2 c_\alpha^2 \simeq s^2 c^2 / (1 + \Delta\rho_t). \quad (\text{H.9})$$

Solving the last equation, we obtain

$$c_\alpha^2 \simeq c^2 \left( 1 + \frac{s^2}{c^2 - s^2} \Delta\rho_t \right), \quad (\text{H.10})$$

$$s_\alpha^2 \simeq s^2 \left( 1 - \frac{c^2}{c^2 - s^2} \Delta\rho_t \right), \quad (\text{H.11})$$

and therefore,

$$\frac{m_W^2}{m_Z^2} \simeq c_\alpha^2 (1 + \Delta\rho_t) \simeq c^2 \left( 1 + \frac{c^2}{c^2 - s^2} \Delta\rho_t \right), \quad (\text{H.12})$$

and

$$\frac{m_W}{m_Z} \simeq c + \frac{3\bar{\alpha}}{32\pi} \frac{c}{(c^2 - s^2)s^2} t. \quad (\text{H.13})$$

Likewise,

$$\begin{aligned} g_\nu^2 &\simeq g_{A_i}^2 \simeq \frac{1}{4} \left( \frac{\bar{\alpha}}{c_\alpha^2 s_\alpha^2} / \frac{\bar{\alpha}}{c^2 s^2} \right) \simeq \frac{1}{4} (1 + \Delta\rho_t) \simeq \\ &\simeq \frac{1}{4} \left( 1 + \frac{3\bar{\alpha}}{16\pi s^2 c^2} t \right). \end{aligned} \quad (\text{H.14})$$

$$\begin{aligned} \frac{g_{V_i}}{g_{A_i}} &\simeq (1 - 4s_\alpha^2) \simeq (1 - 4s^2) + \frac{4c^2 s^2}{c^2 - s^2} \Delta\rho_t = \\ &= (1 - 4s^2) + \frac{3\bar{\alpha}}{4\pi(c^2 - s^2)} t. \end{aligned} \quad (\text{H.15})$$

The corrections proportional to  $m_t^2$  were first pointed out by Veltman [29], who emphasized the appearance of such corrections for a large difference  $m_t^2 - m_b^2$  which violates the isotopic symmetry. In this review, the coefficients in front of the factors  $t$  in equations (H.13)–(H.15) are used as coefficients for normalized radiative corrections  $V_i$ .

## Appendix I

### Explicit form of the functions $T_i(t)$ and $H_i(h)$ .

The equations for  $T_i(t)$  and  $H_i(h)$  are [42], [63]:

$$\begin{aligned}
 & \underline{i = m} \\
 T_m(t) &= \left(\frac{2}{3} - \frac{8}{9}s^2\right) \ln t - \frac{4}{3} + \frac{32}{9}s^2 + \\
 & + \frac{2}{3}(c^2 - s^2)\left(\frac{t^3}{c^6} - \frac{3t}{c^2} + 2\right) \ln \left|1 - \frac{c^2}{t}\right| + \\
 & + \frac{2}{3} \frac{c^2 - s^2}{c^4} t^2 + \frac{1}{3} \frac{c^2 - s^2}{c^2} t + \left[\frac{2}{3} - \frac{16}{9}s^2 - \frac{2}{3}t - \frac{32}{9}s^2 t\right] F_t(t) ; \\
 H_m(h) &= -\frac{h}{h-1} \ln h + \frac{c^2 h}{h-c^2} \ln \frac{h}{c^2} - \frac{s^2}{18c^2} h - \frac{8}{3}s^2 + \\
 & + \left(\frac{h^2}{9} - \frac{4h}{9} + \frac{4}{3}\right) F_h(h) - \\
 & - (c^2 - s^2) \left(\frac{h^2}{9c^4} - \frac{4}{9} \frac{h}{c^2} + \frac{4}{3}\right) F_h\left(\frac{h}{c^2}\right) + \\
 & + (1.1203 - 2.59\delta s^2) ;
 \end{aligned} \tag{I.1}$$

where  $\delta s^2 = 0.23110 - s^2$  (note the sign!).

$$\begin{aligned}
 & \underline{i = A} \\
 T_A(t) &= \frac{2}{3} - \frac{8}{9}s^2 + \frac{16}{27}s^4 - \frac{1 - 2tF_t(t)}{4t-1} + \\
 & + \left(\frac{32}{9}s^4 - \frac{8}{3}s^2 - \frac{1}{2}\right) \left[\frac{4}{3}tF_t(t) - \frac{2}{3}(1+2t)\frac{1-2tF_t(t)}{4t-1}\right] ; \\
 H_A(h) &= \frac{c^2}{1-c^2/h} \ln \frac{h}{c^2} - \frac{8h}{9(h-1)} \ln h + \\
 & + \left(\frac{4}{3} - \frac{2}{3}h + \frac{2}{9}h^2\right) F_h(h) - \left(\frac{4}{3} - \frac{4}{9}h + \frac{1}{9}h^2\right) F'_h(h) - \\
 & - \frac{1}{18}h + [0.7752 + 1.07\delta s^2] .
 \end{aligned} \tag{I.2}$$

$$\begin{aligned}
 & \underline{i = R} \\
 T_R(t) &= \frac{2}{9} \ln t + \frac{4}{9} - \frac{2}{9}(1+11t)F_t(t) ; \\
 H_R(h) &= -\frac{4}{3} - \frac{h}{18} + \frac{c^2}{1-c^2/h} \ln \frac{h}{c^2} +
 \end{aligned} \tag{I.3}$$

$$+(\frac{4}{3}-\frac{4}{9}h+\frac{1}{9}h^2)F_h(h)+\frac{h}{1-h}\ln h+\\+(1.3590+0.51\delta s^2)$$

$$\underline{i=\nu}$$

$$T_\nu(t)=T_A(t)$$

$$H_\nu(h)=H_A(h) \tag{I.4}$$

The functions  $F_t$  and  $F_h$  are the limiting cases of the function  $F(s, m_1, m_2)$ , described in Appendix B. The explicit formulas for  $F_t(t)$  and  $F_h(h)$  are given in Appendix J (equation (J.3)) and Appendix L (equation (L.11)), respectively.

## Appendix J.

### The contribution of heavy fermions to the self-energy of the vector bosons.

Let us give the expressions for the contribution of third-generation quarks  $(t, b)$  to the polarization operators (self-energies) of the vector bosons. We use the following notations:  $t = m_t^2/m_Z^2$ ,  $b = m_b^2/m_Z^2$ , ( $b \ll 1$ ),  $h = m_H^2/m_Z^2$ ,  $\Pi_\gamma(q^2) = \Sigma_{\gamma\gamma}(q^2)/m_Z^2$ ,  $\Pi_{\gamma Z}(q^2) = \Sigma_{\gamma Z}(q^2)/m_Z^2$ ,  $\Pi_W(q^2) = \Sigma_W(q^2)/m_W^2$ .

The dimensional regularization yields the terms

$$\Delta_i = \frac{2}{4-D} - \gamma + \ln 4\pi - \ln \frac{m_i^2}{\mu^2} , \quad (\text{J.1})$$

where  $i = t, b, W, Z, \dots$ ,  $D$  is the variable dimension of spacetime, ( $4 - D = 2\varepsilon$ ,  $\varepsilon \rightarrow 0$ ),  $\gamma = -\Gamma'(1) = 0.577 \dots$  (we follow [80], p. 53).

We begin with an auxiliary function  $F_t(t)$ , obtained as a limiting case of the function  $F(s, m_1, m_2)$  (see Appendix B and [80], p 54; [81], p 88),

$$F_t(t) \equiv F(s = m_Z^2, m_t, m_t) = F(1, t, t) , \quad (\text{J.2})$$

and get, using [80],

$$F_t(t) = \begin{cases} 2[1 - \sqrt{4t-1} \arcsin \frac{1}{\sqrt{4t}}] , & 4t > 1 , \\ 2[1 - \sqrt{1-4t} \ln \frac{1+\sqrt{1-4t}}{\sqrt{4t}}] , & 4t < 1 . \end{cases} \quad (\text{J.3})$$

The asymptotics of  $F_t$  are

$$\begin{aligned} F_t &\simeq \ln t + 2 & t \rightarrow 0 , \\ F_t &\simeq 1/6t + 1/60t^2 & t \rightarrow \infty . \end{aligned} \quad (\text{J.4})$$

Differentiation gives

$$F'_t \equiv m_Z^2 \frac{dF}{dm_Z^2} = -t \frac{d}{dt} F_t = \frac{1-2tF_t}{4t-1} . \quad (\text{J.5})$$

In this Appendix,  $\Pi_i$  stands for the contribution of the doublet  $(t, b)$  to the corresponding polarization operator:

$$\Pi_\gamma(0) = 0 . \quad (\text{J.6})$$

$$\Pi_\gamma(m_Z^2) = \frac{\bar{\alpha}}{\pi} \left[ Q_t^2 \left( \Delta_t + (1+2t)F_t(t) - \frac{1}{3} \right) + Q_b^2 \left( \Delta_b + \frac{5}{3} + \ln b \right) \right] . \quad (\text{J.7})$$

$$\Pi_{\gamma Z}(0) = 0 , \quad (\text{J.8})$$

$$\begin{aligned} \Pi_{\gamma Z}(m_Z^2) &= \frac{\bar{\alpha}}{cs\pi} \left\{ \left( \frac{Q_t}{4} - s^2 Q_t^2 \right) \left( \Delta_t + (1+2t)F_t(t) - \frac{1}{3} \right) - \right. \\ &\quad \left. - \left( \frac{Q_b}{4} + s^2 Q_b^2 \right) \left( \Delta_b + \frac{5}{3} + \ln b \right) \right\} . \end{aligned} \quad (\text{J.9})$$

$$\Pi_W(0) = -\frac{\bar{\alpha}}{4\pi s^2 c^2} \left[ \frac{3}{2} t \Delta_t + \frac{3}{4} t \right] . \quad (\text{J.10})$$

$$\begin{aligned} \Pi_W(m_W^2) &= \frac{\bar{\alpha}}{4\pi s^2} \left\{ \left( 1 - \frac{3t}{2c^2} \right) \Delta_t + \frac{5}{3} - \frac{t}{c^2} - \frac{t^2}{2c^4} - \right. \\ &\quad \left. - \left( 1 - \frac{3t}{2c^2} + \frac{t^3}{2c^6} \right) \ln \left| 1 - \frac{c^2}{t} \right| \right\} . \end{aligned} \quad (\text{J.11})$$

$$\begin{aligned} \Pi_Z(m_Z^2) &= \frac{\bar{\alpha} s^2}{\pi c^2} \left[ Q_t^2 \left( \Delta_t + (1+2t)F_t(t) - \frac{1}{3} \right) + Q_b^2 \left( \Delta_b + \frac{5}{3} + \ln b \right) \right] - \\ &\quad - \frac{\bar{\alpha}}{2\pi c^2} \left[ Q_t \left( \Delta_t + (1+2t)F_t(t) - \frac{1}{3} \right) - Q_b \left( \Delta_b + \frac{5}{3} + \ln b \right) \right] + \\ &\quad + \frac{\bar{\alpha}}{8\pi s^2 c^2} \left[ (2-3t)\Delta_t + (1-t)F_t(t) + \frac{4}{3} + \ln t \right] . \end{aligned} \quad (\text{J.12})$$

$$\Pi_Z(0) = -\frac{\bar{\alpha}}{4\pi s^2 c^2} \left( \frac{3}{2} t \Delta_t \right) . \quad (\text{J.13})$$

$$\begin{aligned} \Sigma'(m_Z^2) &= \frac{\bar{\alpha} s^2}{\pi c^2} \left[ Q_t^2 \left( \Delta_t + F_t - \frac{1}{3} + (1+2t)F'_t \right) + Q_b^2 \left( \Delta_b + \frac{5}{3} + \ln b \right) \right] - \\ &\quad - \frac{\bar{\alpha}}{2\pi c^2} \left[ Q_t \left( \Delta_t + F_t - \frac{1}{3} + (1+2t)F'_t \right) - Q_b \left( \Delta_b + \frac{5}{3} + \ln b \right) \right] + \\ &\quad + \frac{\bar{\alpha}}{8\pi s^2 c^2} \left[ 2\Delta_t + F_t + \frac{4}{3} + \ln t + (1-t)F'_t - 1 \right] = \\ &= \frac{\bar{\alpha} s^2}{\pi s^2} \left[ Q_t^2 \left( \Delta_t + \frac{4+2t}{3(4t-1)} + \frac{1-2t+4t^2}{1-4t} F_t \right) + Q_b^2 \left( \Delta_b + \frac{5}{3} + \ln b \right) \right] - \\ &\quad - \frac{\bar{\alpha}}{2\pi c^2} \left[ Q_t \left( \Delta_t + \frac{4+2t}{3(4t-1)} + \frac{1-2t+4t^2}{1-4t} F_t \right) - Q_b \left( \Delta_b + \frac{5}{3} + \ln b \right) \right] + \\ &\quad + \frac{\bar{\alpha}}{8\pi s^2 c^2} \left[ 2\Delta_t + \ln t + \frac{2+t}{3(4t-1)} + \frac{2t^2+2t-1}{4t-1} F_t \right] . \end{aligned} \quad (\text{J.14})$$

$$\begin{aligned} \Pi_Z(m_Z^2) - \Sigma'_Z(m_Z^2) &= [2tF_t(t) - (1+2t)F'_t(t)] \left[ \frac{\bar{\alpha} s^2}{\pi c^2} Q_t^2 - \frac{\bar{\alpha}}{2\pi c^2} Q_t - \right. \\ &\quad \left. - \frac{\bar{\alpha}}{16\pi s^2 c^2} \right] + \frac{3\bar{\alpha}}{8\pi s^2 c^2} t \Delta_t + \frac{\bar{\alpha}}{16\pi s^2 c^2} (2-3F'_t(t)) + \frac{\bar{\alpha}}{2\pi c^2} Q_b + \frac{\bar{\alpha} s^2}{\pi c^2} Q_b^2 . \end{aligned} \quad (\text{J.15})$$

Substituting the expressions for the polarization operators into the formulas for physical observables, we verify the cancelation of the terms  $\sim \Delta_i$ , and also the terms proportional to  $\ln b$ , since the limit  $m_b \rightarrow 0$  does not produce divergences. It is convenient to get rid of the terms  $\sim \Delta_b$  and  $\sim \ln b$  already in the expression for the polarization operators using equation

$$\Delta_b + \ln(b) = \Delta_t + \ln(t) \quad (\text{J.16})$$

and then making sure that the terms  $\sim \Delta_t$  are indeed eliminated.

Our definition of the wavefunction of the  $Z$ -boson differs in sign from that assumed in [80]; hence the quantity  $\Pi_{\gamma Z}$  we use also differs in sign from the expression given in [80]. With our definition, the interaction of  $Z$  bosons with Weyl fermions is  $-ie\bar{f}\gamma_\mu f(T_3 - Qs^2)Z_\mu$ , and that of photons is  $-ieQ\bar{f}\gamma_\mu fA_\mu$ . The latter vertex coincides with the one given in equations (8)–(9) of [80], while the former differs in sign.

Let us look at the formulas for physical observables.

The quantity  $T_m(t)$  (I.1) is defined as the following combination of polarization operators:

$$\begin{aligned} t + T_m(t) &= \frac{16\pi s^4}{3\bar{\alpha}} \left\{ \frac{c^2}{s^2} [\Pi_Z(m_Z^2) - \Pi_W(m_W^2)] + \right. \\ &\quad \left. + \Pi_W(m_W^2) - \Pi_W(0) - \Pi_\gamma(m_Z^2) \right\}, \end{aligned} \quad (\text{J.17})$$

since  $\Pi_{\gamma Z}(0) = 0$  for fermion loops.

Using equations (J.7), (J.10)–(J.12) we obtain

$$\begin{aligned} t + T_m(t) &= t + \left( \frac{2}{3} - \frac{8}{9}s^2 \right) \ln t - \frac{4}{3} + \frac{32}{9}s^2 + \frac{c^2 - s^2}{3c^2}t + \\ &\quad + \frac{2}{3}(c^2 - s^2) \left( \frac{t^3}{c^6} - \frac{3t}{c^2} + 2 \right) \ln \left| 1 - \frac{c^2}{t} \right| + \frac{2(c^2 - s^2)}{3c^4}t^2 + \\ &\quad + \left( \frac{2}{3} - \frac{16}{9}s^2 - \frac{2}{3}t - \frac{32}{9}s^2t \right) F_t(t) , \end{aligned} \quad (\text{J.18})$$

where we have taken into account that  $\Pi_\gamma(m_Z^2)$  cancels out with the sum of terms proportional to  $Q_t^2$  and  $Q_b^2$  in  $\Pi_Z(M_Z^2)$ . The terms proportional to  $t$  arise from  $\Pi_W(0)$  and  $\Pi_W(M_Z^2)$ ; those proportional to  $t^2$  arise from  $\Pi_W(m_W^2)$ ; the terms proportional to  $\ln t$  and the constants arise from  $\Pi_W(m_W^2)$  and  $\Pi_Z(m_Z^2)$ ;  $\ln |1 - \frac{c^2}{t}|$  corresponds to the threshold  $(t\bar{b})$  in  $\Pi_W(m_W^2)$ , and finally, the only source of terms proportional to  $F_t(t)$  is  $\Pi_Z(m_Z^2)$ .

The infinities in the contribution of the doublet  $(t, b)$  to the observables must cancel each other since the introduction of an additional fermion family into the electroweak theory does not violate its renormalizability.

Substituting the terms proportional to  $\Delta_t$  in (J.17) and taking into account (J.16), we obtain zero:

$$\begin{aligned} \Delta_t \left\{ \frac{c^2}{s^2} \left[ \frac{\bar{\alpha}}{8\pi s^2 c^2} (2 - 3t) - \frac{\bar{\alpha}}{2\pi c^2} - \frac{\bar{\alpha}}{4\pi s^2} \left( 1 - \frac{3t}{2c^2} \right) \right] + \right. \\ \left. + \frac{\bar{\alpha}}{4\pi s^2} \left( 1 - \frac{3t}{2c^2} \right) + \frac{\bar{\alpha}}{4\pi s^2 c^2} \frac{3}{2}t \right\} = 0 . \end{aligned} \quad (\text{J.19})$$

The expression for  $T_A(t)$  is

$$t + T_A(t) = \frac{16\pi s^2 c^2}{3\bar{\alpha}} [\Pi_Z(m_Z^2) - \Sigma'_Z(m_Z^2) - \Pi_W(0)] . \quad (\text{J.20})$$

Using (J.10), (J.12) and (J.14), we have

$$\begin{aligned} t + T_A(t) &= t + \frac{2}{3} - \frac{8}{9}s^2 + \frac{16}{27}s^4 - F'_t + \left( \frac{32}{9}s^4 - \frac{8}{3}s^2 - \frac{1}{2} \right) \times \\ &\quad \times \left( \frac{4}{3}tF_t - \frac{2(1+2t)}{3}F'_t \right) . \end{aligned} \quad (\text{J.21})$$

The terms proportional to  $\Delta_t$  obviously cancel out.  $\Pi_W(0)$  gives a contribution proportional to  $t$ , while all other terms arise from the difference  $\Pi_Z(m_Z^2) - \Sigma'_Z(m_Z^2)$ .

Finally, we look at  $T_R(t)$ :

$$t + T_R(t) = -\frac{16\pi c^2 s^2}{3\bar{\alpha}} \left\{ \frac{(c^2 - s^2)}{cs} \Pi_{Z\gamma}(m_Z^2) + \Pi_\gamma(m_Z^2) - \Pi_Z(m_Z^2) + \Pi_W(0) \right\} . \quad (\text{J.22})$$

The terms proportional to  $Q_t^2$  and  $Q_b^2$  cancel out, so that only  $\Pi_W(0)$  remain, as well as terms that do not contain  $Q_{t,b}^2$  coming from  $\Pi_{Z\gamma}$  and  $\Pi_Z$ . Substituting (J.10), (J.9) and (J.12), we have

$$t + T_R(t) = t + \frac{4}{9} + \frac{2}{9} \ln t - \frac{2}{9} (1 + 11t) F(t) . \quad (\text{J.23})$$

Here  $t$  comes from  $\Pi_W(0)$ , the term proportional to  $F_t(t)$  – from  $\Pi_Z$ , and  $\frac{4}{9} + \frac{2}{9} \ln t$  – from  $\Pi_Z$  and  $\Pi_{Z\gamma}$ .

Cancelation of infinities in (J.22) follows from

$$\Delta_t \left\{ (c^2 - s^2) \frac{\alpha}{4\pi cs} + cs \left[ \frac{\alpha}{2\pi c^2} - \frac{(2 - 3t)\alpha}{8\pi c^2 s^2} - \frac{3t\alpha}{8\pi s^2 c^2} \right] \right\} = 0 . \quad (\text{J.24})$$

## Appendix K.

### The contribution of light fermions to the self-energy of the vector bosons.

The contribution of the doublet of light fermions to the polarization operators is readily obtained using the formulas of the preceding Appendix.

To achieve this,  $\Delta_{Z,W}$  must be substituted into the formulas of Appendix J instead of  $\Delta_q + \ln(m_q/m_{Z,W})^2$ . The physical reason for the absence of terms proportional to the logarithm of the mass of light quarks and leptons is the infrared stability of the quantities that are analyzed in this Appendix. In the formulas (K.4) and (K.7) we use the equality  $Q_u - Q_d = 1$ . The subscripts  $u$  and  $d$  stand for the upper and lower components of the doublet:

$$\Pi_\gamma(0) = 0 \quad . \quad (\text{K.1})$$

$$\Pi_\gamma(m_Z^2) = \frac{N_c \bar{\alpha}}{3\pi} (Q_u^2 + Q_d^2) (\Delta_Z + \frac{5}{3}) \quad . \quad (\text{K.2})$$

$$\Pi_{\gamma Z}(0) = 0 \quad . \quad (\text{K.3})$$

$$\Pi_{\gamma Z}(m_Z^2) = \frac{N_c \bar{\alpha}}{3cs\pi} (\Delta_Z + \frac{5}{3}) [\frac{1}{4} - (Q_u^2 + Q_d^2)s^2] \quad . \quad (\text{K.4})$$

$$\Pi_W(0) = 0 \quad . \quad (\text{K.5})$$

$$\Pi_W(m_W^2) = \frac{N_c \bar{\alpha}}{12\pi s^2} (\Delta_W + \frac{5}{3}); \quad . \quad (\text{K.6})$$

$$\Pi_Z(m_Z^2) = \frac{N_c \bar{\alpha}}{3\pi s^2 c^2} (\Delta_Z + \frac{5}{3}) [\frac{1}{4} - \frac{s^2}{2} + s^4(Q_u^2 + Q_d^2)] \quad . \quad (\text{K.7})$$

$$\Pi_Z(0) = 0 \quad . \quad (\text{K.8})$$

$$\Sigma'_Z(m_Z^2) = \frac{N_c \bar{\alpha}}{3\pi s^2 c^2} (\Delta_Z + \frac{2}{3}) [\frac{1}{4} - \frac{s^2}{2} + s^4(Q_u^2 + Q_d^2)] \quad . \quad (\text{K.9})$$

Formulas (K.1)– K.9) must be used for three lepton doublets  $(\nu_e, e)$ ,  $(\nu_\mu, \mu)$  and  $(\nu_\tau, \tau)$  with  $N_c = 1$  and two quarks doublets  $(u, d)$  and  $(c, s)$  with  $N_c = 3$ .

Substituting equations (K.1) - (K.9) into expressions for physical observables in terms of polarization operators (L.16), (L.20) and (L.24), we arrive at the contributions to the constants  $C_i$  owing to the self-energies.



## Appendix L.

### The contribution of the vector and scalar bosons to the self-energy of the vector bosons.

This Appendix gives formulas for boson contributions to polarization operators that we reproduced from [80], pp 53, 54. (There is a misprint in [80]: the term proportional to  $\Delta_W$  in the expressions for  $\Pi_W(q^2)$  must be multiplied by  $1/3$ ).

The polarization operators in [80] depend on  $c_W$  and  $s_W$  via coupling constants and depend dynamically on the ratio  $m_W/m_Z$ , which arises from Feynman integrals. We substitute everywhere  $c$  for  $c_W$  (and  $m_W/m_Z$ ) and  $s$  for  $s_W$ . In the framework of the one-loop approximation, this substitution is justified. After this substitution, we find expressions for physical observables; ultraviolet divergences of polarization operators cancel out in these expressions.

In the following formulas  $\Pi_i$  denotes only boson contributions to the corresponding polarization operator (all calculations were performed in the 'tHooft–Feynman gauge):

$$\begin{aligned}\Pi_\gamma(m_Z^2) &= -\frac{\bar{\alpha}}{4\pi} \left\{ 3\Delta_W + 2(3 + 4c^2) \left[ 1 - \sqrt{4c^2 - 1} \arcsin\left(\frac{1}{2c}\right) \right] \right\} = \\ &= -\frac{\bar{\alpha}}{4\pi} (3\Delta_W + 1.53) .\end{aligned}\tag{L.1}$$

$$\Pi_\gamma(0) = 0 .\tag{L.2}$$

$$\Pi_{\gamma Z}(0) = -\frac{\bar{\alpha}}{4\pi cs} (2c^2 \Delta_W) = -\frac{c\bar{\alpha}}{2\pi s} \Delta_W .\tag{L.3}$$

$$\begin{aligned}\Pi_{\gamma Z}(m_Z^2) &= -\frac{\bar{\alpha}}{4\pi cs} \left\{ (5c^2 + \frac{1}{6}) \Delta_W \right. \\ &\quad \left. + 2\left(\frac{1}{6} + \frac{13}{3}c^2 + 4c^4\right) \left[ 1 - \sqrt{4c^2 - 1} \arcsin\left(\frac{1}{2c}\right) \right] + \frac{1}{9} \right\} = \\ &= \frac{-\bar{\alpha}}{4\pi} \left( \frac{30c^2 + 1}{6cs} \Delta_W + 3.76 \right) .\end{aligned}\tag{L.4}$$

$$\begin{aligned}\Pi_W(0) &= \frac{\bar{\alpha}}{4\pi s^2} \left[ \left( \frac{s^2}{c^2} - 1 \right) \Delta_W + \frac{3}{4(1 - c^2/h)} \ln \frac{c^2}{h} - \frac{h}{8c^2} + s^2 \right. \\ &\quad \left. + \frac{s^4}{c^2} - \frac{1}{8c^2} - \frac{39}{12} + \left( \frac{s^2}{c^2} + 3 - \frac{17}{4s^2} \right) \ln c^2 \right] \\ &= \frac{\bar{\alpha}}{4\pi s^2} \left[ \left( \frac{s^2}{c^2} - 1 \right) \Delta_W + \frac{3}{4(1 - c^2/h)} \ln \frac{c^2}{h} - \frac{h}{8c^2} + 0.85 \right] .\end{aligned}\tag{L.5}$$

$$\begin{aligned}
\Pi_W(m_W^2) &= \frac{\bar{\alpha}}{4\pi s^2} \left\{ -\left(\frac{25}{6} - \frac{s^2}{c^2}\right) \Delta_W \right. \\
&+ \left[ \frac{s^4}{c^2} - \frac{c^2}{3} \left( \frac{7}{c^2} + 17 - 2\frac{s^4}{c^4} \right) - \frac{1}{6} \left( \frac{1}{2} + \frac{1}{c^2} - \frac{s^4}{2c^4} \right) \right] F_h(1/c^2) \\
&+ \left[ \frac{c^2}{3} \left( \frac{3}{c^2} + 21 \right) - \frac{s^4}{c^2} + \frac{1}{4} \right] \frac{1}{s^2} \ln\left(\frac{1}{c^2}\right) - 3s^2 - \frac{1}{6c^2} + \frac{s^4}{c^2} - \frac{113}{18} \\
&+ \left. \left( 1 - \frac{h}{3c^2} + \frac{h^2}{12c^4} \right) F_h(h/c^2) - 1 - \frac{h}{6c^2} + \frac{3h}{4(c^2 - h)} \ln(h/c^2) \right\} \\
&= \frac{\bar{\alpha}}{4\pi s^2} \left[ \left( \frac{s^2}{c^2} - \frac{25}{6} \right) \Delta_W - 1.76 + \left( 1 - \frac{h}{3c^2} + \frac{h^2}{12c^4} \right) F_h(h/c^2) - \frac{h}{6c^2} \right. \\
&+ \left. \frac{3h \ln(h/c^2)}{4(c^2 - h)} \right]. \tag{L.6}
\end{aligned}$$

$$\begin{aligned}
\Pi_Z(m_Z^2) &= \frac{\bar{\alpha}}{4\pi s^2} \left\{ \left( 7s^2 - \frac{25}{6} + \frac{7s^2}{6c^2} \right) \Delta_W + \frac{73}{36c^2} - \frac{2}{9} + \frac{13}{12c^2} \ln(c^2) \right. \\
&+ \left( 2 + \frac{1+8c^2}{6c^2} (c^2 - s^2)^2 - \frac{20}{3} c^2 (1 + 2c^2) \right) \\
&\times \left[ 1 - \sqrt{4c^2 - 1} \arcsin\left(\frac{1}{2c}\right) \right] \\
&- \frac{1}{c^2} - \frac{h}{6c^2} + \frac{3h}{4c^2(1-h)} \ln h + \left( \frac{1}{c^2} - \frac{h}{3c^2} + \frac{h^2}{12c^2} \right) F_h(h) \Big\} \\
&= \frac{\bar{\alpha}}{4\pi s^2} \left[ \left( 7s^2 - \frac{25}{6} + \frac{7s^2}{6c^2} \right) \Delta_W - 0.58 - \frac{h}{6c^2} + \frac{3h}{4c^2(1-h)} \ln h \right. \\
&+ \left. \left( \frac{1}{c^2} - \frac{h}{3c^2} + \frac{h^2}{12c^2} \right) F_h(h) \right]. \tag{L.7}
\end{aligned}$$

$$\begin{aligned}
\Sigma'_Z(m_Z^2) &= \frac{d\Sigma_Z(s)}{ds} \Big|_{s=m_Z^2} \\
&= \frac{\bar{\alpha}}{4\pi} \left( 3 - \frac{19}{6s^2} + \frac{1}{6c^2} \right) \Delta_W \\
&+ \frac{\bar{\alpha}}{48\pi s^2 c^2} \left\{ \left[ -40c^4 + (c^2 - s^2)^2 \right] 2 \left[ 1 - \sqrt{4c^2 - 1} \arcsin\left(\frac{1}{2c}\right) \right] \right. \\
&+ \left[ 12c^2 + (8c^2 + 1)(c^2 - s^2)^2 - 40c^4(1 + 2c^2) \right] \\
&\times \left[ -1 + \frac{4c^2}{\sqrt{4c^2 - 1}} \arcsin\left(\frac{1}{2c}\right) \right] \\
&+ \left[ 1 - (h - 1)^2 \right] F_h(h) + \left[ 11 - 2h + (1 - h)^2 \right] F'_h(h) \\
&+ \left. \left[ 1 - \frac{1+h}{2(h-1)} \ln h - \frac{1}{2} \ln \frac{h}{c^4} \right] + \frac{2}{3} \left[ 1 + (c^2 - s^2)^2 - 4c^4 \right] \right\}
\end{aligned}$$

$$\begin{aligned}
&= \frac{\bar{\alpha}}{4\pi} \left( 3 - \frac{19}{6s^2} + \frac{1}{6c^2} \right) \Delta_W \\
&+ \frac{\bar{\alpha}}{4\pi s^2 c^2} \left[ \left( 1 - \frac{h}{3} + \frac{h^2}{12} \right) F'_h(h) + \left( \frac{h}{6} - \frac{h^2}{12} \right) F_h(h) \right. \\
&\left. + \frac{h}{12(1-h)} \ln h - 1.67 \right] .
\end{aligned} \tag{L.8}$$

The functions  $F_h(h)$  and  $F'_h(h)$  are defined as

$$F_h(h) \equiv F(s, m_Z, m_H) \big|_{s=m_Z^2} \equiv F(1, 1, h) , \tag{L.9}$$

$$F'_h(h) \equiv s \frac{dF(s, m_Z, m_H)}{ds} \big|_{s=m_Z^2} . \tag{L.10}$$

Using [81], pp 88, we obtain

$$\begin{aligned}
F_h(h) &= 1 + \left( \frac{h}{h-1} - \frac{1}{2}h \right) \ln h + h \sqrt{1 - \frac{4}{h}} \ln \left( \sqrt{\frac{h}{4}} - 1 + \sqrt{\frac{h}{4}} \right) , \quad h > 4, \\
&= 1 + \left( \frac{h}{h-1} - \frac{1}{2}h \right) \ln h - h \sqrt{\frac{4}{h} - 1} \arctan \sqrt{\frac{4}{h} - 1} , \quad h < 4.
\end{aligned} \tag{L.11}$$

If  $h \rightarrow \infty$ ,

$$F_h(h) \approx \frac{1}{2h} - \frac{1}{h^2} \left( 1 + \frac{4}{h^2} \right) \ln h + \frac{5}{3h^2} + \frac{59}{12h^3} . \tag{L.12}$$

If  $h \rightarrow 0$ ,

$$F_h(h) \approx 1 - \pi \sqrt{h} + (1 - \frac{3}{2} \ln h) h . \tag{L.13}$$

Finally, for  $F'_h(h)$  we have

$$\begin{aligned}
F'_h(h) &= -1 + \frac{h-1}{2} \ln h + (3-h) \sqrt{\frac{h}{h-4}} \ln \left( \sqrt{\frac{h-4}{4}} + \sqrt{\frac{h}{4}} \right) , \quad h > 4, \\
&= -1 + \frac{h-1}{2} \ln h + (3-h) \sqrt{\frac{h}{4-h}} \arctan \sqrt{\frac{4-h}{h}} , \quad h < 4.
\end{aligned} \tag{L.14}$$

If  $h \rightarrow \infty$ ,

$$F'_h(h) \approx \frac{1}{2h} - \frac{1}{h^2} \ln(h) . \tag{L.15}$$

All infinities in formulas (L.1)–(L.8) are collected into the factors  $\Delta_W$  by replacing the factors  $\Delta_Z$  and  $\Delta_H$  using the equation

$$\Delta_i = \Delta_j + \ln(m_j^2/m_i^2) .$$

The function  $H_m(h)$  is

$$\begin{aligned}
H_m(h) &= \frac{16\pi s^4}{3\bar{\alpha}} \left( \frac{c^2}{s^2} [\Pi_Z(m_Z^2) - \Pi_W(m_W^2)] + \Pi_W(m_W^2) - \Pi_W(0) \right. \\
&\quad \left. - \Pi_\gamma(m_Z^2) - 2\frac{s}{c}\Pi_{\gamma Z}(0) \right) - \text{div}H_m ,
\end{aligned} \tag{L.16}$$

where  $\text{div}H_m$  denotes the sum of terms, proportional to  $\Delta_W$ , in polarization operators in (L.16). Substituting the finite parts of the formulas for polarization operators from this Appendix, we obtain

$$\begin{aligned}
H_m(h) &= -\frac{h}{h-1} \ln h + \frac{c^2 h}{h-c^2} \ln \frac{h}{c^2} - \frac{s^2}{18c^2} h + \left( \frac{h^2}{9} - \frac{4h}{9} + \frac{4}{3} \right) F_h(h) \\
&\quad - (c^2 - s^2) \left( \frac{h^2}{9c^4} - \frac{4h}{9c^2} + \frac{4}{3} \right) F_h \left( \frac{h}{c^2} \right) + 0.50 ,
\end{aligned} \tag{L.17}$$

where the term proportional to  $F_h(h)$  arises from  $\Pi_Z$ , and the term proportional to  $F_h(h/c^2)$  arises from  $\Pi_W(m_W^2)$ . The term proportional to  $\ln h$  originates from  $\Pi_Z$ , while  $\ln(h/c^2)$  arises both from  $\Pi_W(m_W^2)$  and from  $\Pi_W(0)$ . The term proportional to  $h$  is contained in  $\Pi_Z$ ,  $\Pi_W(m_W^2)$  and  $\Pi_W(0)$  and finally, all four polarization operators make contribution to the constant.

Collecting the terms proportional to  $\Delta_W$  in the polarization operators in (L.16), we get

$$\begin{aligned}
\text{div}H_m &= \frac{16\pi s^4}{3\bar{\alpha}} \Delta_W \left[ \frac{\bar{\alpha}}{4\pi s^2} \frac{c^2}{s^2} \left( 7s^2 - \frac{25}{6} + \frac{7}{6} \frac{s^2}{c^2} + \frac{25}{6} - \frac{s^2}{c^2} \right) \right. \\
&\quad \left. + \frac{\bar{\alpha}}{4\pi s^2} \left( -\frac{19}{6} \right) + \frac{3\bar{\alpha}}{4\pi} + 2\frac{s}{c} \frac{c}{s} \frac{\bar{\alpha}}{2\pi} \right] = \frac{16\pi s^4}{3\bar{\alpha}} \Delta_W \frac{\bar{\alpha}}{\pi s^2} .
\end{aligned} \tag{L.18}$$

Note that the divergent term in  $D$  (see (E.8) and (M.26)) exactly compensates for the divergence in (L.18), which justifies the subtraction of infinity in (L.16).

The function  $H_A(h)$  is expressed in terms of polarization operators as follows:

$$H_A(h) = \frac{16\pi s^2 c^2}{3\bar{\alpha}} [\Pi_Z(m_Z^2) - \Sigma'_Z(m_Z^2) - \Pi_W(0)] - \text{div}H_A . \tag{L.19}$$

Substituting the finite parts of the polarization operators, we obtain

$$\begin{aligned}
H_A(h) &= \frac{hc^2}{h-c^2} \ln \frac{h}{c^2} - \frac{8h}{9(h-1)} \ln(h) + \left( \frac{4}{3} - \frac{2}{3}h + \frac{2}{9}h^2 \right) F_h(h) \\
&\quad - \left( \frac{4}{3} - \frac{4h}{9} + \frac{h^2}{9} \right) F'_h(h) - \frac{h}{18} + 0.78 ,
\end{aligned} \tag{L.20}$$

where  $\ln(h/c^2)$  stems from  $\Pi_W$  and  $\ln h$  stems from  $\Pi_Z$ .  $F_h(h)$  arises both from  $\Pi_Z$  and from  $\Sigma'_Z$ , while the only source of  $F'_h(h)$  is  $\Sigma'_Z$ . The term linear in  $h$  is contained in  $\Pi_Z$  and  $\Pi_W$ , while all three polarization operators contribute to the constant.

Adding up the divergent terms, we have

$$\text{div}H_A = \frac{16}{3} c^2 s^2 \Delta_W = \frac{16\pi s^2 c^2}{3\bar{\alpha}} \Delta_W \frac{\bar{\alpha}}{\pi} . \tag{L.21}$$

Note that the divergent part of  $D_A$  (see (M.18) and (M.14)) is:

$$\text{div} D_A = -\frac{16c^2}{3}\Delta_W = -\frac{16\pi s^2 c^2}{3\bar{\alpha}}\Delta_W \frac{\bar{\alpha}}{\pi s^2} . \quad (\text{L.22})$$

Vertex parts also contain ultraviolet divergences (see Appendix M, eqs.(M.11), (M.15)):

$$\text{div} \tilde{F}_A = -\left(\frac{16}{3}c^2 s^2 - \frac{16}{3}c^2\right)\Delta_W = \frac{16\pi s^2 c^2}{3\bar{\alpha}}\Delta_W \frac{c^2}{s^2} \frac{\bar{\alpha}}{\pi} . \quad (\text{L.23})$$

The sum of terms (L.21), (L.22) and (L.23) equals zero.

Finally, the expression for  $H_R(h)$  is

$$\begin{aligned} H_R(h) = & -\frac{16\pi}{3\bar{\alpha}}c^2 s^2 \left( \frac{(c^2 - s^2)}{cs} \Pi_{Z\gamma}(m_Z^2) + \Pi_\gamma(m_Z^2) - \Pi_Z(m_Z^2) + \Pi_W(0) \right. \\ & \left. + 2\frac{s}{c}\Pi_{\gamma Z}(0) \right) - \text{div} H_R(h) . \end{aligned} \quad (\text{L.24})$$

Collecting the finite parts of the polarization operators, we find

$$H_R(h) = -\frac{h}{18} + \frac{c^2}{1 - c^2/h} \ln \frac{h}{c^2} + \left(\frac{4}{3} - \frac{4}{9}h + \frac{1}{9}h^2\right)F_h(h) + \frac{h}{1 - h} \ln h + 0.03 . \quad (\text{L.25})$$

The term proportional to  $F_h(h)$  stems from  $\Pi_Z$ , just as  $\ln h$  does.  $\Pi_W$  generates the term  $\sim \ln(h/c^2)$ . The term linear in  $h$  is contained both in  $\Pi_Z$  and in  $\Pi_W$ , and all polarization operators with the exception of  $\Pi_{\gamma Z}(0)$  contribute to the constant.

Adding up the divergent parts of the polarization operators in (L.24), we obtain

$$\begin{aligned} \text{div} H_R(h) = & \Delta_W \left[ \frac{2}{9}(c^2 - s^2)(1 + 30c^2) + 4c^2 s^2 + \frac{4}{3}c^2 \left( 7s^2 - \frac{25}{6} \right. \right. \\ & \left. \left. + \frac{7}{6}\frac{s^2}{c^2} \right) + \frac{4}{3}c^2 - \frac{4}{3}s^2 + \frac{16}{3}c^2 s^2 \right] = \Delta_W (8c^2 - \frac{16}{3}c^4) . \end{aligned} \quad (\text{L.26})$$

Taking into account the divergent term in  $D$ , yielding (see eqs. (E.8), (M.14), (M.18))

$$\text{div} D_R = -\frac{16}{3}c^2 \Delta_W , \quad (\text{L.27})$$

and in  $\tilde{F}_R$  (see Appendix M, equations (M.11) and (M.22)),

$$\text{div} \tilde{F}_R = \left(\frac{16}{3}c^4 - \frac{8}{3}c^2\right)\Delta_W , \quad (\text{L.28})$$

we confirm that divergences cancel out in expressions for  $R$ .

## Appendix M

### The vertex parts of $F_{Af}$ and $F_{Vf}$ and the constants $C_i$ .

This Appendix collects the vertex functions that form a part of one-loop electroweak corrections to  $Z \rightarrow \nu\bar{\nu}$ ,  $Z \rightarrow l^+l^-$ ,  $Z \rightarrow u\bar{u}$ ,  $c\bar{c}$ ,  $d\bar{d}$  and  $s\bar{s}$  decays. In the case of the  $Z \rightarrow b\bar{b}$  decay, a  $t$ -quark can propagate in the loop, so vertex corrections are not reducible to numbers but are functions of  $m_t$  (see Appendix N).

The finite parts of vertex functions are given in [80], pp 29, 30. The corresponding expressions depend on  $c_W(s_W)$  and  $m_W/m_Z$ . In the framework of the one-loop approximation, we replace  $c_W$  and  $m_W/m_Z$  with  $c$ , and  $s_W$  with  $s$ . For this reason, while vertex functions in [80] depend on  $m_t$ ,  $m_H$  and the new physics, ours are numbers (see also Appendix L).

This Appendix also gives the infinite parts absent from [80] and required for testing whether the infinities in physical observables do cancel out.

We begin with the  $Z \rightarrow \nu\nu$  decay:

$$\begin{aligned} F_\nu &\equiv F_{V\nu} = F_{A\nu} = \\ &= \frac{\bar{\alpha}}{4\pi} \frac{1}{4cs} \left[ \frac{1}{4c^2s^2} \Lambda_2(m_Z^2, m_Z) + \frac{2s^2 - 1}{2s^2} \Lambda_2(m_Z^2, m_W) + \frac{3c^2}{s^2} \Lambda_3(m_Z^2, m_W) \right] \end{aligned} \quad (M.1)$$

For the decay to a pair of charged leptons or quarks we have

$$F_{Vf} = \frac{\bar{\alpha}}{4\pi} \left[ v_f(v_f^2 + 3a_f^2) \Lambda_2(m_Z^2, m_Z) + F_L^f \right] , \quad (M.2)$$

$$F_{Af} = \frac{\bar{\alpha}}{4\pi} \left[ a_f(3v_f^2 + a_f^2) \Lambda_2(m_Z^2, m_Z) + F_L^f \right] , \quad (M.3)$$

where  $a_l = a_d = -1/4sc$ ,  $a_u = 1/4sc$ ,  $v_f = (T_3^f - 2Q^f s^2)/2sc$  ( $T_3^l = T_3^d = -1/2$ ,  $T_3^u = 1/2$ ,  $Q^l = -1$ ,  $Q^d = -1/3$ ,  $Q^u = 2/3$ ). The functions  $F_L^f$  are

$$F_L^l = \frac{1}{8s^3c} \Lambda_2(m_Z^2, m_W) - \frac{3c}{4s^3} \Lambda_3(m_Z^2, m_W) , \quad (M.4)$$

$$F_L^u = -\frac{1 - \frac{2}{3}s^2}{8s^3c} \Lambda_2(m_Z^2, m_W) + \frac{3c}{4s^3} \Lambda_3(m_Z^2, m_W) , \quad (M.5)$$

$$F_L^d = \frac{1 - \frac{4}{3}s^2}{8s^3c} \Lambda_2(m_Z^2, m_W) + \frac{3c}{4s^3} \Lambda_3(m_Z^2, m_W) . \quad (M.6)$$

For calculating  $F_{Vf}$  and  $F_{Af}$  we need to determine the values of three constants:  $\Lambda_2(m_Z^2, m_W)$ ,  $\Lambda_2(m_Z^2, m_Z)$  and  $\Lambda_3(m_Z^2, m_W)$ :

$$\begin{aligned} \Lambda_2(m_Z^2, m_W) &= -\frac{7}{2} - 2c^2 - (2c^2 + 3) \ln(c^2) \\ &+ 2(1 + c^2)^2 \left[ \ln c^2 \ln \left( \frac{1 + c^2}{c^2} \right) - Sp \left( -\frac{1}{c^2} \right) \right] , \end{aligned} \quad (M.7)$$

where we have used  $m_W/m_Z = c$ ;  $Sp(x)$  is the Spence function:

$$Sp(x) = - \int_0^1 \frac{dt}{t} \ln(1 - xt) , \quad Sp(-1) = -\frac{\pi^2}{12} . \quad (\text{M.8})$$

Using (M.7) and (M.8), we find

$$\Lambda_2(m_Z^2, m_Z) = -\frac{7}{2} - 2 - 8Sp(-1) . \quad (\text{M.9})$$

Finally,

$$\begin{aligned} \Lambda_3(m_Z^2, m_W) &= \frac{5}{6} - \frac{2}{3}c^2 + \frac{2}{3}(2c^2 + 1)\sqrt{4c^2 - 1} \arctan \frac{1}{\sqrt{4c^2 - 1}} \\ &\quad - \frac{8}{3}c^2(c^2 + 2) \left( \arctan \frac{1}{\sqrt{4c^2 - 1}} \right)^2 . \end{aligned} \quad (\text{M.10})$$

The expressions for divergent parts of the vertex functions, describing the coupling of the  $Z$ -boson to the leptons, are

$$div F_\nu = \frac{\bar{\alpha}}{8\pi} \frac{c}{s^3} \Delta_W , \quad div F_{Vl} = div F_{Al} = -div F_\nu \quad (\text{M.11})$$

We switch to the calculation of the constants  $C_i$ . We begin with definitions. According to [42],

$$V_\nu(t, h) = t + T_\nu(t) + H_\nu(h) + L_\nu + D_\nu + \tilde{F}_\nu , \quad (\text{M.12})$$

The value of  $L_\nu$  represents the contribution of leptons and light quarks to the polarization operators of the vector bosons and can be easily obtained from the formulas of Appendix K for polarization operators:

$$L_\nu = 4 - 8s^2 + \frac{304}{27}s^4 . \quad (\text{M.13})$$

$D_\nu$  originates from the box and vertex electroweak corrections to the  $\mu$ -decay [23]. The expression for  $D$  see in Appendix E, equation (E.8). For  $D_\nu$  we have

$$D_\nu = -\frac{16\pi s^2 c^2}{3\bar{\alpha}} \left( D - \frac{\bar{\alpha}}{\pi s^2} \Delta_W \right) . \quad (\text{M.14})$$

Finally,

$$\tilde{F}_\nu = \frac{128\pi s^3 c^3}{3\bar{\alpha}} F_\nu . \quad (\text{M.15})$$

Comparing (M.12) and (58), we arrive at the expressions for  $C_\nu$  whose ingredients are now all determined:

$$C_\nu = L_\nu + D_\nu + \tilde{F}_\nu \quad (\text{M.16})$$

Let us switch to  $V_A$ :

$$V_A(t, h) = t + T_A(t) + H_A(h) + L_A + D_A + \tilde{F}_A . \quad (\text{M.17})$$

The expressions for  $L_A$  and  $D_A$  are already there:

$$L_A = L_\nu \quad , \quad D_A = D_\nu \quad ; \quad (\text{M.18})$$

the formula for the vertex function is

$$\tilde{F}_A = -\frac{128\pi s^3 c^3}{3\bar{\alpha}} F_{Al} \quad . \quad (\text{M.19})$$

Finally,

$$C_A = L_A + D_A + \tilde{F}_A \quad (\text{M.20})$$

We now move to  $V_R$ :

$$V_R(t, h) = t + T_R(t) + H_R(h) + L_R + D_R + \tilde{F}_R \quad , \quad (\text{M.21})$$

where  $D_R = D_\nu$  and  $L_R = 0$  since  $\Pi_W(m_W^2)$  is absent from the ratio  $g_V/g_A$ . For  $\tilde{F}_R$  we have

$$\tilde{F}_R = \frac{16\pi(c^2 - s^2)cs}{3\bar{\alpha}} [-F_{Vl} + (1 - 4s^2)F_{Al}] \quad , \quad (\text{M.22})$$

and the formula for  $C_R$  is

$$C_R = L_R + D_R + \tilde{F}_R \quad . \quad (\text{M.23})$$

We end this Appendix with formulas for  $C_m$ :

$$C_m = L_m + D_m \quad , \quad (\text{M.24})$$

$$L_m = 4(c^2 - s^2) \ln c^2 \quad , \quad (\text{M.25})$$

$$D_m = -\frac{16\pi s^4}{3\bar{\alpha}} \left( D - \frac{\bar{\alpha}}{\pi s^2} \Delta_W \right) \quad . \quad (\text{M.26})$$



## Appendix N.

### The functions $\phi(t)$ and $\delta\phi(t)$ in the $Z \rightarrow b\bar{b}$ decay.

For the function  $\phi(t)$  we use the expansion from [57]:

$$\begin{aligned} \phi(t) = \frac{3-2s^2}{2s^2c^2} \left\{ t + c^2[2.88\ln\frac{t}{c^2} - 6.716 + \right. \\ + \frac{1}{t}(8.368c^2\ln\frac{t}{c^2} - 3.408c^2) + \frac{1}{t^2}(9.126c^4\ln\frac{t}{c^2} + 2.26c^4) + \\ \left. + \frac{1}{t^3}(4.043c^6\ln\frac{t}{c^2} + 7.41c^6) + \dots \right\} , \end{aligned} \quad (\text{N.1})$$

and for  $\delta\phi(t)$  we use the leading approximation calculated in [58] and [53]:

$$\delta\phi(t, h) = \frac{3-2s^2}{2s^2c^2} \left\{ -\frac{\pi^2}{3} \left( \frac{\hat{\alpha}_s(m_t)}{\pi} \right) t + \frac{1}{16s^2c^2} \left( \frac{\bar{\alpha}}{\pi} \right) t^2 \tau_b^{(2)} \left( \frac{h}{t} \right) \right\} , \quad (\text{N.2})$$

where the function  $\tau_b^{(2)}$  is tabulated in Table 4 for  $m_H/m_t < 4$ . For  $m_H/m_t > 4$  we use the expansion [53]

$$\begin{aligned} \tau_b^{(2)}\left(\frac{h}{t}\right) = \frac{1}{144} [311 + 24\pi^2 + 282\ln r + 90\ln^2 r - 4r(40 + 6\pi^2 + 15\ln r + 18\ln^2 r) + \\ + \frac{3}{100} r^2 (24209 - 6000\pi^2 - 45420\ln r - 18000\ln^2 r)], \end{aligned} \quad (\text{N.3})$$

where  $r = t/h$ . For  $m_t = 175$  GeV and  $m_H = 300$  GeV

$$\tau_b^{(2)} = 1.245 \ .$$

## Appendix O.

### The $\delta_2 V_i$ corrections.

The corrections  $\delta_2 V_i \sim \bar{\alpha} \hat{\alpha}_s$  arise from gluon exchanges in quark electroweak loops [54] (see also [82]). For two generations of light quarks ( $q = u, d, s, c$ ) we have

$$\delta_2^q V_m(t, h) = 2 \cdot \left[ \frac{4}{3} \left( \frac{\hat{\alpha}_s(m_Z)}{\pi} \right) (c^2 - s^2) \ln c^2 \right] = \left( \frac{\hat{\alpha}_s(m_Z)}{\pi} \right) (-0.377) \quad (\text{O.1})$$

$$\delta_2^q V_A(t, h) = \delta_2^q V_\nu(t, h) = 2 \cdot \left[ \frac{4}{3} \left( \frac{\hat{\alpha}_s(m_Z)}{\pi} \right) (c^2 - s^2 + \frac{20}{9} s^4) \right] = \left( \frac{\hat{\alpha}_s(m_Z)}{\pi} \right) (1.750) \quad (\text{O.2})$$

$$\delta_2^q V_R(t, h) = 0 \quad (\text{O.3})$$

The result of calculations for the third generation is obtained in the form of fairly complicated functions of the  $t$ -quark mass:

$$\begin{aligned} \delta_2^t V_m(t, h) = & \frac{4}{3} \left( \frac{\hat{\alpha}_s(m_t)}{\pi} \right) \left\{ t A_1 \left( \frac{1}{4t} \right) + \left( 1 - \frac{16}{3} s^2 \right) t V_1 \left( \frac{1}{4t} \right) + \left( \frac{1}{2} - \frac{2}{3} s^2 \right) \ln t \right. \\ & \left. - 4 \left( 1 - \frac{s^2}{c^2} \right) \times t F_1 \left( \frac{c^2}{t} \right) - 4 \frac{s^2}{c^2} t F_1(0) \right\} \end{aligned} \quad (\text{O.4})$$

$$\begin{aligned} \delta_2^t V_A(t, h) = \delta_2^t V_\nu(t, h) = & \frac{4}{3} \left( \frac{\hat{\alpha}_s(m_t)}{\pi} \right) \left\{ t A_1 \left( \frac{1}{4t} \right) - \frac{1}{4} A_1' \left( \frac{1}{4t} \right) + \right. \\ & \left. + \left( 1 - \frac{8}{3} s^2 \right)^2 \left[ t V_1 \left( \frac{1}{4t} \right) - \frac{1}{4} V_1' \left( \frac{1}{4t} \right) \right] + \left( \frac{1}{2} - \frac{2}{3} s^2 + \frac{4}{9} s^4 \right) - 4t F_1(0) \right\} \end{aligned} \quad (\text{O.5})$$

$$\delta_2^t V_R(t, h) = \frac{4}{3} \left( \frac{\hat{\alpha}_s(m_t)}{\pi} \right) \left\{ t A_1 \left( \frac{1}{4t} \right) - \frac{5}{3} t V_1 \left( \frac{1}{4t} \right) - 4t F_1(0) + \frac{1}{6} \ln t \right\}, \quad (\text{O.6})$$

where

$$\hat{\alpha}_s(m_t) = \frac{\hat{\alpha}_s(M_Z)}{1 + \frac{23}{12\pi} \hat{\alpha}_s(M_Z) \ln t} \quad (\text{O.7})$$

Note that  $\delta_2 V_i$  are independent of  $m_H$ . The functions  $V_1(r)$ ,  $A_1(r)$  and  $F_1(x)$  have rather complex form and were calculated in [54]. Their expansions for sufficiently small values of arguments are (we have added cubic terms to the expansions from [54]):

$$V_1(r) = r \left[ 4\zeta(3) - \frac{5}{6} \right] + r^2 \frac{328}{81} + r^3 \frac{1796}{25 \times 27} + \dots \quad (\text{O.8})$$

$$A_1(r) = \left[ -6\zeta(3) - 3\zeta(2) + \frac{21}{4} \right] + r \left[ 4\zeta(3) - \frac{49}{18} \right] + r^2 \frac{689}{405} + r^3 \frac{3382}{7 \times 25 \times 27} + \dots \quad (\text{O.9})$$

$$\begin{aligned} F_1(x) = & \left[ -\frac{3}{2}\zeta(3) - \frac{1}{2}\zeta(2) + \frac{23}{16} \right] + x \left[ \zeta(3) - \frac{1}{9}\zeta(2) - \frac{25}{72} \right] + \\ & + x^2 \left[ \frac{1}{8}\zeta(2) + \frac{25}{3 \times 64} \right] + x^3 \left[ \frac{1}{30}\zeta(2) + \frac{5}{72} \right] + \dots \end{aligned} \quad (\text{O.10})$$

where  $\zeta(2) = \pi^2/6$ ,  $\zeta(3) = 1.2020569\dots$

Adding up the contributions (O.4)–(O.6) and using the expansions (O.8)–(O.10), we obtain (up to terms  $\sim 1/t^3$ ):

$$\delta_2^t V_m(t, h) = \left(\frac{\hat{\alpha}_s(m_t)}{\pi}\right) \left[-2.86t + 0.46 \ln t - 1.540 - \frac{0.68}{t} - \frac{0.21}{t^2}\right] = \frac{\hat{\alpha}_s(m_t)}{\pi} (-11.67) \quad (\text{O.11})$$

$$\delta_2^t V_A(t, h) = \delta_2^t V_\nu(t, h) = \left(\frac{\hat{\alpha}_s(m_t)}{\pi}\right) \left[-2.86t + 0.493 - \frac{0.19}{t} - \frac{0.05}{t^2}\right] = \frac{\hat{\alpha}_s(m_t)}{\pi} (-10.10) \quad (\text{O.12})$$

$$\delta_2^t V_R(t, h) = \left(\frac{\hat{\alpha}_s(m_t)}{\pi}\right) \left[-2.86t + 0.22 \ln t - 1.513 - \frac{0.42}{t} - \frac{0.08}{t^2}\right] = \frac{\hat{\alpha}_s(m_t)}{\pi} (-11.88). \quad (\text{O.13})$$

These formulas hold for  $m_t > m_Z$ . In the region  $m_t < m_Z$  we either set  $\delta_2^t V_i = 0$  or make use of the massless limit  $\delta_2^t V_i = \frac{1}{2} \delta_2^q V_i$ . In any case this region gives negligible contribution to the global fit.

## Appendix P.

### The $\delta_5 V_i$ corrections.

In the second order of weak interactions quadratic dependence on the mass of the higgs boson is given by expressions [83]:

$$\delta_5 V_m = \frac{\bar{\alpha}}{24\pi} \left( \frac{m_H^2}{m_Z^2} \right) \times \frac{0.747}{c^2} = 0.0011 \quad (\text{P.1})$$

$$\delta_5 V_A = \delta_5 V_\nu = \frac{\bar{\alpha}}{24\pi} \left( \frac{m_H^2}{m_Z^2} \right) \times \frac{1.199}{s^2} = 0.0057 \quad (\text{P.2})$$

$$\delta_5 V_R = -\frac{\bar{\alpha}}{24\pi} \left( \frac{m_H^2}{m_Z^2} \right) \frac{c^2 - s^2}{s^2 c^2} \times 0.973 = -0.0032 \quad . \quad (\text{P.3})$$

The numerical evaluations above were made with  $m_H = 300$  GeV.

## References

- [1] S. L. Glashow, Nucl. Phys. **22**, 579(1961); Rev. Mod. Phys. **52**, 539 (1980).  
 S. Weinberg, Phys. Rev. Lett. **19**, 1264 (1967); Rev. Mod. Phys. **52**, 515 (1980).  
 A. Salam, in Elementary Particle Theory, edited by N. Svartholm (Almqvist and Wiksells, Stockholm, 1968), p. 367; Rev. Mod. Phys. **52**, 525 (1980).
- [2] S. L. Glashow, J. Iliopoulos, and L. Maiani, Phys. Rev. **D2**, 1285 (1970).
- [3] For memoirs on the early history of QCD see  
 M. Gell-Mann ‘Quarks, Color and QCD’ in ‘QCD – 20 Years Later’, Ed. P. M. Zerwas and H. A. Kastrup, vol. 1, p. 3. World Scientific, 1993. See also general review of QCD, with bibliography, in other contributions to this two-volume publication.  
 W. Marciano and H. Pagels, Phys. Reports **36C**, 137 (1978).
- [4] M. B. Voloshin, K. A. Ter-Martirosyan, Theory of Gauge Interactions of Elementary Particles. Energoatomizdat, Moscow, 1984;  
 C. Itzykson, J.-B. Zuber, Quantum Field Theory. McGraw-Hill. New York, 1980; vol 1 and 2, Mir, Moscow, 1984;  
 E. D. Commins, P. H. Bucksbaum, Weak interactions of leptons and quarks. Cambridge University Press, 1983; Energoatomizdat, Moscow, 1987;  
 L. B. Okun, Leptons and Quarks. Nauka, Moscow, 1990.  
 2nd edition, revised;  
 L. B. Okun, Leptons and Quarks. North Holland, Amsterdam, 1982; 2nd revised edn, Nauka, Moscow, 1990;  
 P. Ramon, Field Theory, Moscow, Mir, 1984;  
 P. Ramon, Field Theory, A Modern Primer, Benjamin/Cummings, Inc., London, 1981;  
 A. A. Slavnov, L. D. Faddeev, Introduction to Quantum Theory of Gauge Fields, 2nd edition, Nauka, Moscow, 1988;  
 F. Halzen, A. Martin, Quarks and Leptons. John Wiley, New York, 1984;  
 Mir, Moscow, 1987;  
 T. -P. Cheng and L. -F. Li, Gauge Theory of Elementary Particle Physics. Clarendon Press, 1984;  
 H. Georgi, Weak Interactions and Modern Particle Theory. Benjamin/Cummings. Inc. 1984;  
 K. Gottfried, V. Weisskopf, Concept of Particle Physics, v. I and II, Oxford University press. New York. 1986.
- [5] N. Cabibbo, Phys. Rev. Lett. **10**, 531 (1963).
- [6] H. J. Hasert et al. , Phys. Lett. **B46**, 138 (1973);  
 H. J. Hasert et al. , Nucl. Phys. **B73**, 1 (1974).
- [7] L. M. Barkov, M. S. Zolotarev, Pis'ma v ZhETF **26**, 544 (1978);  
 L. M. Barkov, M. S. Zolotarev, Phys. Lett. **B85**, 308 (1979).
- [8] C. Y. Prescott et al. , Phys. Lett. **B77**, 347 (1978);  
 C. Y. Prescott et al. , Phys. Lett. **B84**, 524 (1979).

- [9] J. J. Aubert et al. , Phys. Rev. Lett. **33**, 1404(1974);  
J. E. Augustin et al. , Phys. Rev. Lett. **33**, 1406(1974).
- [10] M. Kobayashi, T. Maskawa, Progr. Theor. Phys. **49**, 652 (1973).
- [11] M. L. Perl et al. , Phys. Rev. Lett. **35**, 1489(1975);  
M. L. Perl et al. , Phys. Lett. **63B**, 460(1976).
- [12] S. W. Herb et al. , Phys. Rev. Lett. **39**, 252 (1977).
- [13] CDF Collaboration: F. Abe et al. , Phys. Rev. Lett. **74**, 2626 (1995).
- [14] D0 Collaboration: S. Abachi et al. , Phys. Rev. Lett. **74**, 2632 (1995).
- [15] G. 't Hooft, Nucl. Phys. **B33**, 173 (1971);  
G. 't Hooft, Nucl. Phys. **B35**, 167 (1971).
- [16] H. D. Politzer, Phys. Rev. Lett. **30**, 1346 (1973);  
D. J. Gross, F. Wilczek, Phys. Rev. Lett. **30**, 1343 (1973);  
D. J. Gross, F. Wilczek, Phys. Rev. **D8**, 3633 (1973).
- [17] G. Arnison et al. , Phys. Lett. **B122**, 103 (1983);  
M. Banner et al. , Phys. Lett. **B122**, 476 (1983);  
G. Arnison et al. , Phys. Lett. **B126**, 398 (1983);  
P. Bagnaia et al. , Phys. Lett. **B129**, 310 (1983).
- [18] Reports of the working group on precision calculations for the  $Z$  resonance, CERN 95-03, Eds. D. Bardin, W. Hollik, G. Passarino.
- [19] H. E. Haber and R. Hempfling, Phys. Rev. Lett. **66**, 1815 (1991).
- [20] Y. Okada, M. Yamaguchi, and T. Yanagida, Progr. Theor. Phys. **85**, 1 (1991);  
J. Ellis, G. Ridolfi, and F. Zwirner, Phys. Lett. **B257**, 83 (1991).
- [21] Physics at LEP, Eds. J. Ellis, R. Peccei, CERN 86-02 (1986).
- [22] Z Physics at LEP1, Eds. G. Altarelli, R. Kleiss, C. Verzegnassi, CERN 89-08 (1989).
- [23] A. Sirlin, Phys. Rev. **D22**, 971 (1980).
- [24] S. M. Berman, A. Sirlin, Ann. Phys. (N. Y. ) **20**, 20 (1962);  
A. Sirlin, Rev. Mod. Phys. **50**, 573 (1978).
- [25] E. S. Abers, D. A. Dicus, R. Norton and H. Quinn, Phys. Rev. **167**, 1461 (1968);  
A. Sirlin, Nucl. Phys. **B71**, 29 (1974);  
A. Sirlin, Rev. Mod. Phys. **50**, 573 (1978).

- [26] H. Burkhardt, F. Jegerlehner, G. Penso and C. Verzegnassi, Z. Phys. **C43**, 497 (1989).  
 B. V. Geshkenbein, V. L. Morgunov, Phys. Lett. **340**, 185 (1994), preprint ITEP 19-95, Moscow, 1995;  
 M. I. Vysotsky, R. B. Nevzorov, A. V. Novikov, Pis'ma v ZhETF **60**, 392 (1994), Yad. Fiz. **59**, 721 (1996);  
 M. L. Swartz, preprint SLAC-PUB-6710, 1994;  
 A. D. Martin, D. Zeppenfeld, preprint MAD/PH/855, 1994;  
 S. Eidelman, F. Jegerlehner, Z. Phys. **C67**, 585 (1995).
- [27] D. Yu. Bardin, V. A. Dokuchaeva, Nucl. Phys. **B246**, 221 (1984);  
 S. Sarantakos, A. Sirlin, W. J. Marciano, Nucl. Phys. **B227**, 84 (1983);  
 R. G. Stuart, Z. Phys. **C34**, 445 (1987).
- [28] A. Sirlin, W. J. Marciano, Phys. Rev. **D22**, 2695 (1980); Nucl. Phys. **B189**, 442 (1981);  
 J. F. Wheeler, C. H. Llewellynn-Smith, Nucl. Phys. **B208**, 27 (1982) (errata: *ibid*, **B226**, 547 (1983));  
 C. H. Llewellynn-Smith, Nucl. Phys. **B228**, 205 (1983);  
 D. Yu. Bardin, V. A. Dokuchaeva, Preprint JINR, E2-86-260, 1986, Dubna, unpublished.
- [29] M. Veltman, Nucl. Phys. **B123**, 89 (1977);  
 M. Veltman, Acta Phys. Pol. , **B8**, 475 (1977).
- [30] CDHS: H. Abramowitz et al. , Phys. Rev. Lett. **57**, 298 (1986).  
 A. Blondel et al. , Z. Phys. **C45**, 361 (1990).
- [31] CHARM Collaboration, J. V. Allaby et al. , Phys. Lett. **B177**, 446 (1986).
- [32] CCFR: C. G. Arroyo et al. , Phys. Rev. Lett. **72**, 3452 (1994).
- [33] UA2: Alitti et al. , Phys. Lett. **B276**, 354 (1992).
- [34] CDF: A. Abe et al. , Phys. Rev. Lett. **65**, 2243 (1990).
- [35] Review of Particle Properties, Phys. Rev. **D50** (1994) No. 3, part 1;  
 J.Erler, P.Langacker, *ibid.*, p. 1304.
- [36] K. Hagiwara, Talk at LP95 (10-15 Aug. 1995, Beijing).  
 P. B. Renton, Talk at Europhysics Conference, Brussell, 1995.
- [37] CHARM II Collaboration, Phys. Lett. **B281**, 159 (1992).  
 CHARM II Collaboration, Phys. Lett. **B259**, 499 (1991).
- [38] V. Novikov, L. Okun, M. Vysotsky, Phys. Lett. **B298**, 453 (1993).
- [39] CHARM II Collaboration, Phys. Lett. **B320**, 203 (1994).
- [40] I. B. Khriplovich, Parity Nonconservation in Atomic Phenomena. Nauka, Moscow, 1988 (2nd edition);  
 S. A. Blundell, W. R. Johnson, and J. Sapirstein, Phys. Rev. Lett. **65**, 1411 (1990);  
 M. A. Bouchiat, L. Pottier, Science **234**, 1203 (1986);  
 J. Rosner, W. Marciano, Phys. Rev. Lett. **65**, 2963 (1990).

- [41] B. W. Lynn and M. E. Peskin, report SLAC-PUB-3724 (1985) unpublished.  
B. W. Lynn, M. E. Peskin and R. G. Stuart, in: Physics at LEP, report CERN 86-02 (CERN Geneva).
- [42] V. Novikov, L. Okun, M. Vysotsky, Nucl. Phys. **B397**, 35 (1993).
- [43] S. Willenbrock and G. Valencia, Phys. Lett. **B259**, 573 (1991);  
A. Leike, T. Riemann, J. Rose, Phys. Lett. **B273**, 513 (1991);  
L3 Collaboration (O. Adriani et al. ), Phys. Lett. **B315**, 494 (1993);  
H. Veltman, Z. Phys. **C62**, 35 (1994).
- [44] The LEP Collaborations ALEPH, DELPHI, L3, OPAL and the LEP Electroweak Working Group, preprint CERN-PPR/95-172 (1995).
- [45] S. G. Gorishny, A. L. Kataev and S. A. Larin, Phys. Lett. **B259** (1991) 144;  
L. R. Surguladze and M. A. Samuel, Phys. Rev. Lett. **66** (1991) 560.
- [46] K. G. Chetyrkin, J. H. Kühn, Phys. Lett. **B248** (1990) 359;  
K. G. Chetyrkin, J. H. Kühn and A. Kwiatkowski, Phys. Lett. **282** (1992) 221.
- [47] K. G. Chetyrkin and A. Kwiatkowski, Phys. Lett. **B305** (1993) 288;  
K. G. Chetyrkin and A. Kwiatkowski, Phys. Lett. **B319** (1993) 307;  
K. G. Chetyrkin, Phys. Lett. **B307** (1993) 169;
- [48] S. A. Larin, T. van Ritberger, J. A. M. Vermaseren, Phys. Lett. **B320** (1994) 159;  
K. G. Chetyrkin, O. V. Tarasov, Phys. Lett. **B327** (1994) 114.
- [49] A. L. Kataev, Phys. Lett. **287** (1992) 209.
- [50] N. Gray et al. , Z. Phys. **C48** (1990) 673;  
L. R. Surguladze, Univ. of Oregon Report No. OITS543 hep-ph/9405325 (1994).
- [51] G. Altarelli, Proceedings of the Int. Europhysics Conf. on High Energy Physics, Marseille 1993; Eds. J. Carr and M. Perrotet (Edition Frontieres 1994);  
L. Okun, *ibid.* ;  
D. Schildknecht, *ibid.*
- [52] M. Vysotsky, proceedings of the XXVII Int. Conference on High Energy Physics, Glasgow 1994, Eds. P. J. Bussey and I. G. Knowles (Institute of Physics Publishing).
- [53] R. Barbieri, M. Beccaria, P. Ciafaloni, G. Curci, A. Vicere, Phys. Lett. **B288**, 95 (1992);  
Nucl. Phys. **B409**, 105 (1993);  
J. Fleisher, O. V. Tarasov, F. Jegerlehner, preprint BI-TP-93/24 and PSI-PR-93-14 (1993); Phys. Lett. **B319**, 249 (1993);  
F. Halzen, B. A. Kniehl, M. L. Stong, Z. Phys. **C58**, 119 (1993);  
R. Barbieri, P. Ciafaloni, A. Strumia, Phys. Lett. **B317**, 381 (1993).



- [54] A. Djouadi, C. Verzegnassi, Phys. Lett. **B195**, 265 (1987);  
B. A. Kniehl, Nucl. Phys. **B347**, 86 (1990);  
F. Halzen and B. A. Kniehl, Nucl. Phys. **B353**, 517 (1991).
- [55] V. Novikov, L. Okun, M. Shifman, A. Vainshtein, M. Voloshin and V. Zakharov, Phys. Rep. **41C**, 1 (1978).
- [56] K. G. Chetyrkin, J. H. Kühn, M. Steinhauser, Phys. Lett. **B351**, 331 (1995); Phys. Rev. Lett. **75**, 3394 (1995);  
L. Avdeev, J. Fleisher, S. Mikhailov, O. Tarasov, Phys. Lett. **B336**, 560 (1994); **B349**, 597 (1995) (E).
- [57] A. A. Akhundov, D. Yu. Bardin, T. Riemann, Nucl. Phys. **B276**, 1 (1986);  
J. Bernabeu, A. Pich, A. Santamaria, Phys. Lett. **B200**, 569 (1986);  
W. Beenaker, W. Hollik, Z. Phys. **C40**, 141 (1988).
- [58] J. Fleisher, F. Jegerlehner, P. Raczka, O. V. Tarasov, Phys. Lett. **B293**, 437 (1992);  
C. Buchalla, A. Buras, Nucl. Phys. **B398**, 285 (1990);  
G. Degrassi, Nucl. Phys. **B407**, 271 (1993);  
K. G. Chetyrkin, A. Kwiatkowski, M. Steinhauser, Mod. Phys. Lett. **A8**, 2785 (1993).
- [59] D. Bardin et al. , program ZFITTER 4. 9, Nucl. Phys. **B351**, 1 (1991); Z. Phys. **C44**, 493 (1989); Phys. Lett. **B255**, 290 (1991); CERN-TH. 6443-92 (1992).
- [60] J. Ellis, G. Fogli, Phys. Lett. **213B**, 189, 526 (1988); **232B**, 139 (1989);  
J. Ellis, G. Fogli, Phys. Lett. **B249**, 543 (1990).
- [61] W. Hollik, Fortschr. Phys. **38**, 3, 165 (1990);  
M. Consoli, W. Hollik and F. Jegerlehner: Proceedings of the Workshop on Z physics at LEP I, CERN Report 89-08 Vol. I, 7;  
G. Burgers, F. Jegerlehner, B. Kniehl and J. H. Kühn: CERN Report 89-08 Vol. I, 55.
- [62] G. Montagna, O. Nicrosini, G. Passarino, F. Piccinini and R. Pittau, Nucl. Phys. **B401**, 3 (1993);  
G. Montagna, O. Nicrosini, G. Passarino, F. Piccinini and R. Pittau, program TOPAZO, Comput. Phys. Commun. **76**, 328 (1993).
- [63] V. Novikov, L. Okun, A. Rozanov, M. Vysotsky, preprint ITEP 19-95, CPPM-1-95.  
[http://cppm.in2p3.fr./lepton/intro\\_lepton.html](http://cppm.in2p3.fr./lepton/intro_lepton.html)
- [64] A. Caner, talk at the Conference "Les Recontres de Physique de la Vallee d'Aoste", March 3-9, 1996.
- [65] M. Narain, talk at the Conference "Les Recontres de Physique de la Vallee d'Aoste", March 3-9, 1996.
- [66] M. Boulware, D. Finnell, Phys. Rev. **D44**, 2054 (1991);  
G. L. Kane, C. Kolda, J. D. Wells, Phys. Lett. **B338**, 219 (1994).

- [67] M. Shifman, Minnesota preprint TPI-MINN-95/32-T(1995).
- [68] M. Voloshin, Int. J. Mod. Phys. **A10**, 2865 (1995).
- [69] P. Z. Quintas et al. , Phys. Rev. Lett. **71**, 1307 (1993);  
 J. P. Berge et al. , Z. Phys. **C49**, 187 (1991);  
 M. Arneodo et al. , Phys. Lett. **B309**, 222 (1993) 222;  
 K. Bazizi, S. J. Wimpenny, UCR/DIS/91-02;  
 M. Virchaux, A. Milsztajn, Phys. Lett. **274**, 221 (1992).
- [70] M. Kobel, DESY-F31-91-03.
- [71] M. Voloshin, Preprint ITEP-21-1980, 1980;  
 M. Voloshin, Doctor of Science Theses, Moscow, 1988.
- [72] M. Peskin, T. Takeuchi, Phys. Rev. Lett. **65**, 964 (1990); Phys. Rev. **D46**, 381 (1992).
- [73] G. Altarelli, R. Barbieri, S. Jadach, Nucl. Phys. **B369**, 3 (1992).
- [74] Interim Report on the Physics Motivations for an Energy Upgrade of LEP 2, preprint CERN-TH/95-151, CERN-PPE/95-78.
- [75] L. D. Landau, A. A. Abrikosov, I. M. Khalatnikov, Doklady Akad. Nauk SSSR, **95**, 497; 1177 (1954);  
 M. Gell-Mann and F. Low, Phys. Rev. **95** 1300 (1954).
- [76] V. Novikov, L. Okun, M. Vysotsky, Phys. Lett. **B324**, 89 (1994).
- [77] R. Nevzorov, A. Novikov, Yad. Fiz. **59**, 540 (1996).
- [78] V. A. Novikov, L. B. Okun and M. I. Vysotsky, Mod. Phys. Lett. **A9**, 1489 (1994).
- [79] S. A. Larin, T. Van Ritberger, J. A. M. Vermaseren, Phys. Lett. **B320**, 159 (1994);  
 K. G. Chetyrkin, O. V. Tarasov, Phys. Lett. **B327**, 114 (1994).
- [80] Physics at LEP1, Eds. G. Altarelli, R. Kleiss, C. Verzegnassi (CERN 89-08, Geneva, 1989), v. 1, p. 7. Conveners: M. Consoli, W. Hollik. Working Group: F. Jegerlehner.
- [81] Physics at LEP1, Eds. G. Altarelli, R. Kleiss, C. Verzegnassi (CERN 89-08, Geneva, 1989), v. 1, p. 57. Conveners: G. Burgers, F. Jegerlehner. Working Group: B. Kniehl, J. Kühn.
- [82] M. I. Vysotsky, N. A. Nekrasov, V. A. Novikov, L. B. Okun, Yad. Fiz. **57**, 883 (1994).
- [83] J. Van der Bij, M. Veltman, Nucl. Phys. **B231**, 205 (1984);  
 J. J. Van der Bij, Nucl. Phys. **B248**, 141 (1984).